Internship position:

Solving large sparse linear systems with mixed precision domain decomposition methods

Internship level: Master 2

Duration: 6 months approximately

Location: LIP6 and LJLL laboratories, Sorbonne Université, 4 place Jussieu, Paris 5e

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Salary: on the basis of CNRS internship allowances (27.30 euros/working day and reimbursement of part of transport costs)

Scientific context:
Solving sparse linear systems is one of the fundamental problems of scientific computing. Despite the computing power of modern supercomputers, solving very large systems (with billions of unknowns) remains beyond the reach of traditional methods. Domain decomposition (DD) methods can deal with such large sizes by dividing the global problem into smaller local problems that are each solved independently and in parallel. A global phase ensures the continuity of the solution. However, DD methods require various linear algebra kernels (local preconditioners, eigenvalue problems) that must be accurate to ensure rapid convergence, but must also remain inexpensive to build, in order to maintain the scalability on very large problems. Solving large sparse linear systems in a way that is both frugal and reliable therefore constitutes one of the major current challenges in the field.

Main objectives:
The objective of this internship is to develop mixed precision DD methods, which exploit different levels of precision in order to best optimize their resource consumption. Modern computing architectures (CPUs and GPUs) have several precisions implemented in hardware, notably double (64 bits), single (32 bits), and half (16 bits) precisions. Computations carried out in low precision (32 or even 16 bits) are much faster and save memory space and energy. However, most applications in scientific computing, and in particular those targeted by this internship, require a final accuracy equivalent to 64 bits. The objective is therefore to develop mixed precision methods which only reduce the precision at certain well-chosen locations. This objective is divided into three axes. The first is to determine from a mathematical point of view, through a theoretical error analysis, the places where the precision can be reduced without impacting the final quality of the solution. The second axis consists of the experimental implementation of these strategies in the HPDDM software [3] to exploit modern parallel computing architectures. Finally, the third axis focuses on evaluating the potential of these methods for various realistic physical applications modeled with FreeFEM scripts: elasticity, fluid mechanics, porous flows, etc. This internship will therefore require particular attention to both computer science aspects (efficient implementation of

methods) but also mathematical aspects (rigorous control of the error introduced, theoretical guarantees of robustness).

Work environnement:
The internship will take place at Laboratoire d'Informatique de Paris 6 (LIP6) and in collaboration with Laboratoire Jacques-Louis Lions (LJLL). Both laboratories are located on the Jussieu campus of Sorbonne Université.

Continuation with a PhD thesis:
The internship could lead to a PhD position, in the context of the national project NUMPEX lead by CNRS, CEA, and Inria, whose goal is to design and develop the software stack that will be used by the future exascale computers with applications to climate, the energy transition, health, AI and the industry.

References to go deeper:

[1] V. Dolean, P. Jolivet, F. Nataf. (2015). *An introduction to domain decomposition methods: algorithms, theory, and parallel implementation*. Society for Industrial and Applied Mathematics.
[2] N. Higham, T. Mary. *Mixed precision algorithms in numerical linear algebra*, https://hal.archives-ouvertes.fr/hal-03537373
[3] P. Jolivet, F. Hecht, F. Nataf, C. Prud'Homme. (2013). *Scalable domain decomposition preconditioners for heterogeneous elliptic problems*. In Proceedings of the International Conference on High Performance Computing, Networking, Storage and Analysis.