

Internship offer:

# Optimization of the representation of results in interval arithmetic

**Internship level:**

Master's 2nd year or end of engineering studies

**Duration:**

4 to 6 months

**Location:**

Laboratoire LIP6, Sorbonne Université, 4 place Jussieu, Paris, France

**Supervisors:**

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**Internship context:**

The goal of this internship is to contribute to proposing efficient methods and tools for controlling the numerical reliability of large-scale simulations.

In mathematics and computer science, interval arithmetic [2] is a method of computation that involves manipulating intervals, as opposed to numbers (e.g., integers or floating-point numbers), with the aim of obtaining rigorous results. This approach allows for the bounding of rounding errors and, consequently, the development of numerical methods that provide reliable results. In interval arithmetic, a real number  $x$  is represented by a pair of floating-point numbers ( $x_{inf}$ ,  $x_{sup}$ ). Stating that  $x$  is represented by this pair means that  $x$  belongs to the interval  $[x_{inf}, x_{sup}]$ . This is referred to as the "inf-sup" notation. There is another way to represent intervals, where an interval is defined by its center and radius. It is denoted as  $\langle c, r \rangle$  with  $r > 0$ , representing the interval  $[c - r, c + r]$ . This is known as the "center-radius" notation.

**Tasks to be carried out:**

Currently, it is necessary to use two numbers to encode an interval. The objective of this internship is to define a "compressed" format for storing intervals. To do this, we can draw inspiration from the FP-ANR format [1], which allows the representation of both a number and the associated rounding error without any additional memory overhead. The primary goal is to analyze the impact of this compressed format on the assessment of numerical quality in interval arithmetic.

A new interval arithmetic library based on this compressed format will then be developed. Its performance will be compared to that of conventional interval libraries that use two numbers to represent results.

### **Required skills:**

- C programming, Linux
- Knowledge of floating-point arithmetic principles

### **Internship allowance:**

Based on CNRS internship allowances

(140,70 euros per week as of January 1, 2023, with partial reimbursement of transportation expenses)

### **References**

- [1] David Defour. FP-ANR: A representation format to handle floating-point cancellation at run-time, 25th IEEE Symposium on Computer Arithmetic (ARITH'23), June 2018. <https://hal-lirmm.ccsd.cnrs.fr/lirmm-01549601>
- [2] Ramon E. Moore, R. Baker Kearfott, and Michael J. Cloud. Introduction to interval analysis. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2009.