Precision auto-tuning and control of accuracy in high performance simulations

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Introduction

Floating-point arithmetic:

Sign	Exponent	Mantissa

Various floating-point formats:

	#bits			
	Mantissa (p)	Exp.	Range	$u = 2^{-p}$
bfloat16 (half)	8	8	$10^{\pm 38}$	$\approx 4 \times 10^{-3}$
fp16 (half)	11	5	$10^{\pm 5}$	$\approx 5 \times 10^{-4}$
fp32 (single)	24	8	$10^{\pm 38}$	$\approx 6 \times 10^{-8}$
fp64 (double)	53	11	$10^{\pm 308}$	$\approx 1 \times 10^{-16}$
fp128 (quad)	113	15	$10^{\pm 4932}$	$\approx 1 \times 10^{-34}$

$\searrow$ precision:

- $\searrow$ execution time ☺
- $\searrow$ volume of results exchanged ☺
- $\nearrow$ energy efficiency ☺

energy consumption proportional to p^2

energy ratio	
fp64/fp32	$\approx$ 5
fp32/fp16	$\approx$ 5
fp32/bfloat16	$\approx$ 9

- But computed results may be invalid because of rounding errors ☹

Outline

In this talk we aim at answering the following questions.

1. How to control the numerical quality of floating-point results?
2. How to determine automatically the suitable format for each variable?

Rounding error analysis

Several approaches

- Interval arithmetic
  - guaranteed bounds for each computed result
  - the error may be overestimated
  - specific algorithms
  - ex: INTLAB [Rump'99]
- Static analysis
  - no execution, rigorous analysis, all possible input values taken into account
  - not suited to large programs
  - ex: FLUCTUAT [Goubault & al.'06], FLDLib [Jacquemin & al.'19]
- Probabilistic approach
  - estimates the number of correct digits of any computed result
  - can be used in HPC programs
  - requires no algorithm modification
  - ex: CADNA [Chesneaux'90], VERIFICARLO [Denis & al.'16], VERROU [Févotte & al.'17]

Stochastic arithmetic [Vignes'04]

Classic arithmetic

$A \oplus B \longrightarrow R$

R =3.14237654356891

Stochastic arithmetic

Random rounding

$A_1 \oplus B_1 \longrightarrow R_1$

$A_2 \oplus B_2 \longrightarrow R_2$

$A_3 \oplus B_3 \longrightarrow R_3$

R_1 =**3.14**1354786390989

R_2 =**3.14**3689456834534

R_3 =**3.14**2579087356598

- each operation executed 3 times with a random rounding mode

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- each operation executed 3 times with a random rounding mode
- number of correct digits in the results estimated using Student's test with the confidence level 95%

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- each operation executed 3 times with a random rounding mode
- number of correct digits in the results estimated using Student's test with the confidence level 95%
- operations executed synchronously
  - ⇒ detection of numerical instabilities
    Ex: `if (A>B)` with `A-B` numerical noise
  - ⇒ optimization of stopping criteria

The CADNA library

cadna.lip6.fr

- implements stochastic arithmetic for C/C++ or Fortran codes
- provides stochastic types (3 floating-point variables and an integer)
  `half_st` `float_st` `double_st` `quad_st`
- all operators and mathematical functions overloaded
  $\Rightarrow$ few modifications in user programs
- support for MPI, OpenMP, GPU, vectorised codes
- in one CADNA execution: accuracy of any result, list of numerical instabilities
- overhead: $4\times$ memory, $\approx 10\times$ time

The SAM library

www-pequan.lip6.fr/~jezequel/SAM

SAM (Stochastic Arithmetic in Multiprecision) [Graillat & al.'11]

- implements stochastic arithmetic in arbitrary precision (based on MPFR[1])

  mp_st stochastic type

[1] www.mpfr.org

The SAM library

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SAM (Stochastic Arithmetic in Multiprecision) [Graillat & al.'11]

- implements stochastic arithmetic in arbitrary precision (based on MPFR[1])

  mp_st stochastic type

- recent improvement: control of operations mixing different precisions

  Ex: mp_st<23> A; mp_st<47> B; mp_st<35> C;

$$C = A \oplus B$$

35 bits 23 bits 47 bits

⇒ accuracy estimation on FPGA

[1] www.mpfr.org

An example without/with CADNA

Computation of $P(x,y) = 9x^4 - y^4 + 2y^2$ [Rump'83]

```
#include <iostream>
using namespace std;
double rump(double x, double y) {
  return 9.0*x*x*x*x - y*y*y*y + 2.0*y*y;
}
int main() {
  cout.precision(15);
  cout.setf(ios::scientific,ios::floatfield);
  double x, y;
  x = 10864.0;
  y = 18817.0;
  cout<<"P1="<<rump(x, y)<< endl;
  x = 1.0/3.0;
  y = 2.0/3.0;
  cout<<"P2="<<rump(x, y)<< endl;
  return 0;
}
```

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  return 0;
}
```

P1=2.00000000000000e+00

P2=8.02469135802469e–01

```
#include <iostream>

using namespace std;
double   rump(double   x, double   y) {
  return 9.0*x*x*x*x-y*y*y*y+2.0*y*y;
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int main() {
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```
#include <iostream>
#include <cadna.h>
using namespace std;
double   rump(double   x, double   y) {
  return 9.0*x*x*x*x-y*y*y*y+2.0*y*y;
}
int main() {
  cout.precision(15);
  cout.setf(ios::scientific,ios::floatfield);
  cadna_init(-1);
  double   x, y;
  x=10864.0; y=18817.0;
  cout<<"P1="<<rump(x, y)<<endl;
  x=1.0/3.0; y=2.0/3.0;
  cout<<"P2="<<rump(x, y)<<endl;
  cadna_end();
  return 0;
}
```

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#include <iostream>
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}
```

```
#include <iostream>
#include <cadna.h>
using namespace std;
double_st rump(double_st x, double_st y) {
  return 9.0*x*x*x*x-y*y*y*y+2.0*y*y;
}
int main() {
  cout.precision(15);
  cout.setf(ios::scientific,ios::floatfield);
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  return 0;
}
```

Results with CADNA

only correct digits are displayed

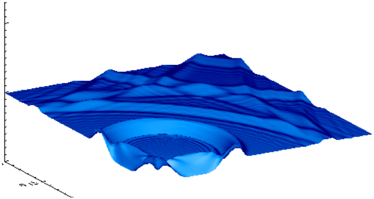
CADNA_C software
Self-validation detection: ON
Mathematical instabilities detection: ON
Branching instabilities detection: ON
Intrinsic instabilities detection: ON
Cancellation instabilities detection: ON

P1= @.0 (no correct digits)
P2= 0.802469135802469E+000

There are 2 numerical instabilities
2 LOSS(ES) OF ACCURACY DUE TO CANCELLATION(S)

Numerical validation of a shallow-water (SW) simulation on GPU

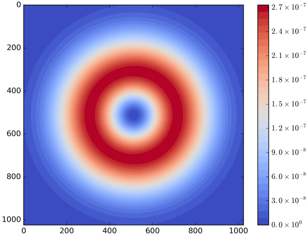
- Simulation of the evolution of water height and velocities in a 2D oceanic basin
- CUDA GPU code in double precision



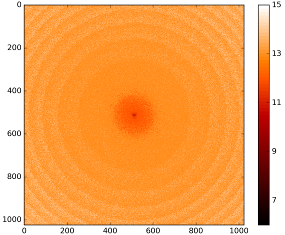
- Focusing on an eddy evolution: 20 time steps (12 hours of simulated time) on a 1024×1024 grid

SW eddy simulation with CADNA-GPU

At the end of the simulation:



Square of water velocity in $m^2.s^{-2}$



Number of correct digits estimated by CADNA-GPU

- at eddy center: great accuracy loss due to cancellations
- point at the very center: 9 digits lost
  ⇒ **no correct digits in single precision**
- fortunately, velocity values close to zero at eddy center
  → negligible impact on the output
  → **satisfactory overall accuracy**

Related tools

Tools related to CADNA available on cadna.lip6.fr

- CADNAIZER
  - automatically transforms C codes to be used with CADNA
- CADTRACE
  - identifies the instructions responsible for numerical instabilities

Other numerical validation tools based on result perturbation

- VERIFICARLO [Denis & al.'16] based on LLVM
- VERROU [Févotte & al.'17] based on Valgrind, no source code modification ☺

asynchronous approach: 1 complete run → 1 result, no accuracy analysis during the run

Numerical validation... and then?

Can we use reduced or mixed precision
to improve performance and energy efficiency?

- mixed precision linear algebra algorithms
  - matrix multiplication,
  - LU and QR matrix factorizations,
  - iterative refinement,
  - Krylov solvers,
  - least squares problems
- precision autotuning

Precision autotuning

- floating-point autotuning tools that intend to deal with large codes:
  - Precimonious [Rubio-Gonzàlez & al.'13]
    - source modification with LLVM
  - CRAFT [Lam & al.'13]
    - binary modifications on the operations
  - ADAPT [Menon & al.'18]
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  They rely on comparisons with the highest precision result.

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float: $P =$ 2.571784e+29

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float: $P =$2.571784e+29

double: $P =$1.17260394005318

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float: $P =$2.571784e+29
double: $P =$1.17260394005318
quad: $P =$1.17260394005317863185883490452018

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float:	$P =$ 2.571784e+29
double:	$P =$ 1.17260394005318
quad:	$P =$ 1.17260394005317863185883490452018
exact:	$P \approx$ -0.827396059946821368141165095479816292

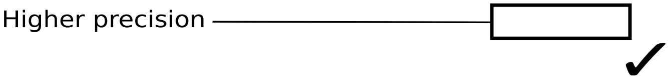
PROMISE (PRecision OptiMISE) [Graillat & al.'19]

promise.lip6.fr

- provides a mixed precision code (half, single, double, quad) taking into account a required accuracy
- uses CADNA to validate a type configuration
- uses the Delta Debug algorithm [Zeller'09] to search for a valid type configuration with a mean complexity of $O(n\log(n))$ for n variables.

Searching for a valid configuration with 2 types

Method based on the Delta Debug algorithm [Zeller'09]



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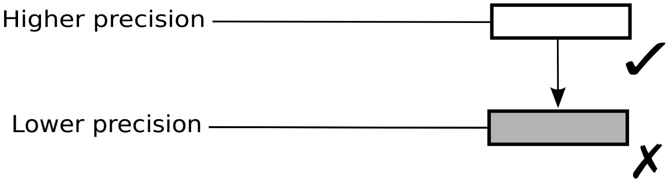

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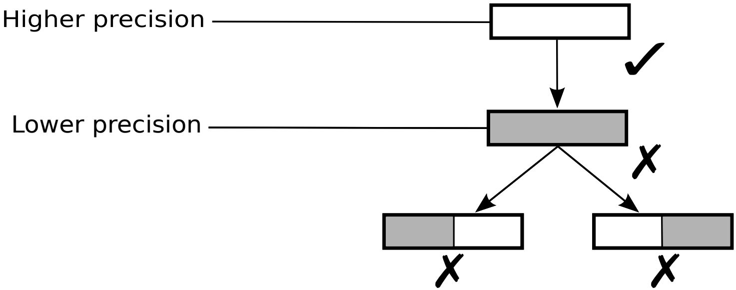

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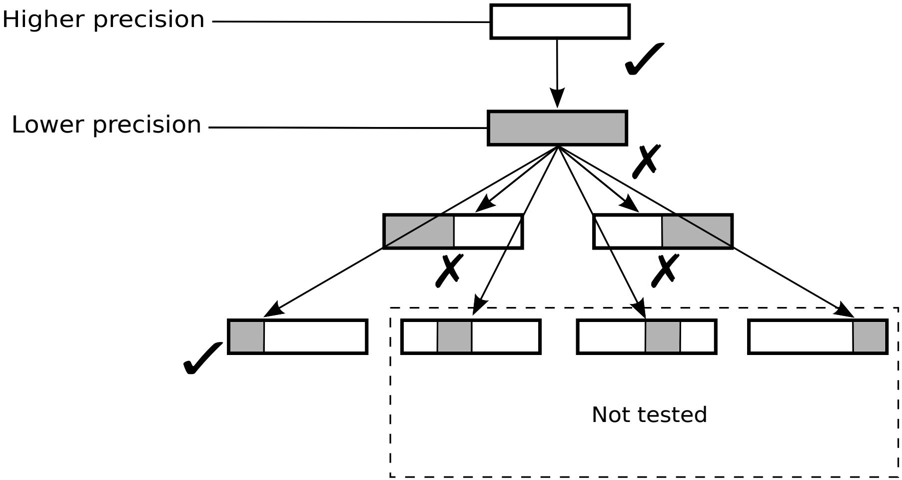

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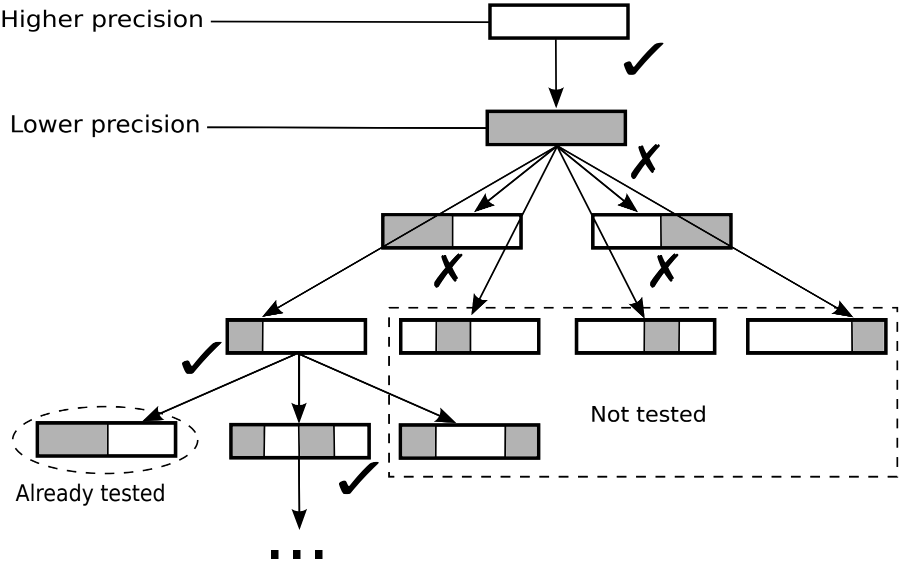
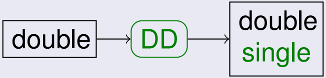


Searching for a valid type configuration

PROMISE with 2 types (ex: double & single precision)

From a code in double, the Delta Debug (DD) algorithm finds which variables can be relaxed to single precision.



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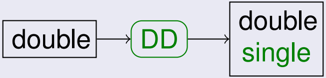
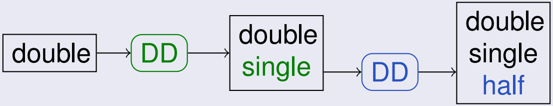


PROMISE with 3 types (ex: double, single & half precision)

The Delta Debug algorithm is applied twice.



Precision autotuning using PROMISE

MICADO: code simulating nuclear cores, developed by EDF (French electricity supplier)

- neutron transport iterative solver
- 11,000 C++ code lines

# Digits	# double - # float	Speed up	memory gain
10	19-32	1.01	1.00
8	18-33	1.01	1.01
6	13-38	1.20	1.44
5 4	0-51	1.32	1.62

- Speedup, memory gain w.r.t. the double precision version
- Speed-up up to 1.32 and memory gain 1.62
- Mixed precision approach successful: speed-up 1.20 and memory gain 1.44

Conclusion/Perspectives

To optimize precision and so improve performance

- numerical validation tools such as CADNA
- precision autotuning tools such as PROMISE
- mixed precision algorithms

Perspectives

- floating-point autotuning in arbitrary precision
- combine mixed precision algorithms and floating-point autotuning

Funded PhD offers to carry out such perspectives
see `http://www.lip6.fr/Fabienne.Jezequel`

Thanks to the CADNA/SAM/PROMISE contributors:
Julien Brajard, Romuald Carpentier, Jean-Marie Chesneaux, Patrick Corde, Pacôme Eberhart, François Févotte, Pierre Fortin, Stef Graillat, Thibault Hilaire, Sara Hoseininasab, Jean-Luc Lamotte, Baptiste Landreau, Bruno Lathuilière, Romain Picot, Jonathon Tidswell, Su Zhou, ...

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Thank you for your attention!