Tight interval inclusions with compensated algorithms

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Keywords: interval arithmetic, directed rounding, compensated algorithms, error-free transformations, floating-point arithmetic, numerical validation, rounding errors, summation algorithms, dot product, Horner scheme

Compensated algorithms consist in computing the rounding errors of individual operations and then adding them later to the computed result. This makes it possible to increase the accuracy of the computed result efficiently. Computing the rounding error of an individual operation is possible through the use of a so-called *error-free transformation*. In this talk, we show that it is possible to use compensated algorithms for having tight interval inclusions. We study compensated algorithms for summation [1], dot product [1] and polynomial evaluation [2]. We prove that the use of directed rounding makes it possible to get narrow inclusions with compensated algorithms. This is due to the fact that error-free transformations are no more exact but still sufficiently accurate to improve and guarantee the numerical quality of the results [3, 4, 5].

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