# Minimal-Precision Computing for High-Performance, Energy-Efficient, and Reliable Computations





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Input:

C code with MPFR

(and MPLAPACK)

**Precision-Optimizer** 

(with PROMISE

and CADNA/SAM)

C code with MPFR

(optimized)

C code with MPFR +

other fast accurate

methods

Compilation and

Execution

on CPU/GPU

\* All authors contributed equally to this research.

# Introduction

In numerical computations, the precision of floating-point computations is a key factor to determine the performance (speed and energy-efficiency) as well as the reliability (accuracy and reproducibility). However, the precision generally plays a contrary role for both. Therefore, the ultimate concept for maximizing both at the same time is the minimal-precision computation through precision-tuning, which adjusts the optimal precision for each operation and data. Several studies have been already conducted for it so far, but the scope of those studies is limited to the precision-tuning alone. In this study, we propose a more broad concept of the minima-precision computing with precision-tuning, involving both hardware and software stack.

## Minimal-precision computing

Minimal-precision computing is both reliable (aka robust) and sustainable as it ensures the requested accuracy of the result as well as is energy-efficient.

Performance can be improved through the minimal-precision as well as fast numerical libraries and accelerators (FPGA and GPU)

## Energy-Efficient

Through the minimal-precision as well as energy efficient hardware acceleration with FPGA and GPU

## Reliable

To ensure the requested accuracy, the precisiontuning is processed based on numerical validation, guaranteeing also reproducibility

#### Comprehensive

tuned code, combining heterogeneous hardware and hierarchal software stack.

# Realistic

realized by combining available in-house technologies

Red: Components developed by us

#### (1) Precision-tuning with numerical validation based on stochastic arithmetic Rounding-errors can be estimated stochastically with a reasonable cost

- (for details, see "A. Stochastic Arithmetic Tools" at the bottom left General scheme applicable for any floating-point computations
- PROMISE [17] (based on a stochastic arithmetic library, **CADNA** [18]), Verrou [19], etc.

#### Issues on existing studies:

- Some existing methods (e.g., Precimonious [20], GPUMixer [22]) are not based on any verification / validation
- No adaptation for heterogeneous systems with FPGA
- The other validation / verification methods:
- > Theoretical error analyses: classic approach based on theory, but not general (analysis is needed for each numerical
- method) and not easy if the code is huge and complex Interval computations provides guaranteed, but may be overestimated bounds, or a special algorithm is needed for each

numerical method, not well suited for large codes

#### High-performance

General

improve performance & energy-efficiency

High-precision arithmetic: binary128 (intel, gcc), QD [1],

Accurate sum/dot: AccSum/Dot [5], Ozaki-scheme [6], etc.

Numerical libraries: MPLAPACK [7], QPEigen [8], QPBLAS

[9], XBLAS [10], ReproBLAS [11], **ExBLAS** [12], **OzBLAS** 

Numerical verification or validation may be more important

MPFR [2], ARPREC [3], CAMPARY [4], etc.

Correct-rounding may be over-accurate and slow

Issues on existing studies:

than bit-level reproducibility

Lack of precision-tuning method

Our scheme is applicable for any floatingpoint computations. It contributes to low development cost and sustainability (easy maintenance and system portability)

## We propose a total system from the precision-tuning to the execution of the

Our system can be

# **Available Components**

#### (2) Arbitrary-precision libraries and fast (3) Field-Programmable Gate Array (FPGA) accurate numerical libraries with High-Level Synthesis (HLS) • FPGA enables us to implement any operations on Reduced-/mixed-precision with FP16/FP32/FP64 enables us to

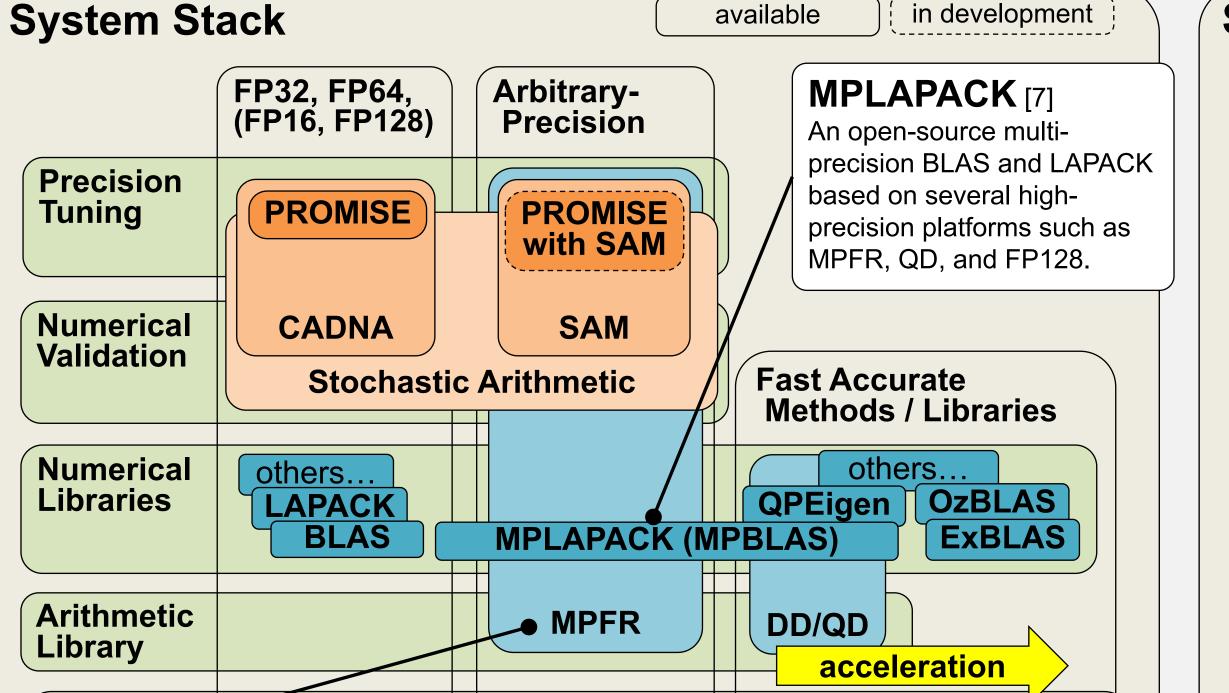
- hardware, including arbitrary-precision operations High-precision libraries and fast accurate computation methods HLS enables us to use FPGAs through existing programming languages such as C/C++ and OpenCL have been developed for reliable & reproducible computation • FPGA can be used to perform arbitrary-precision
  - computations on hardware efficiently (high-performance and energy-efficient)
  - Compilers: SPGen [14], Nymble [15], etc.
  - Custom floating-point operation generator: FloPoCo [16], etc.

#### Issues on existing studies:

 Lack of precision-tuning method Adaptation for HPC as a heterogeneous platform is still at new stage

# **Our System**





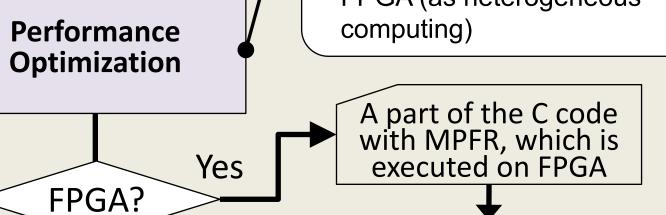
Hardware CPU MPFR [2] Heterogeneous System A C library for multiple

(arbitrary) precision GPU GPU **SPGen** floating-point for arbitrary-prec. Nymble computations on CPUs FloPoCo [16] Compiler & **FPGA** Tools for FPGA An open-source floating-point

# • The Precision-Optimizer determines the minimal floatingpoint precisions, which need to achieve the desired accuracy

#### Performance **Optimization** At this stage, if possible to speedup some parts of the code with some other accurate

- computation methods than MPFR those parts are replaced with them The required-accuracy must be
- taken into account • If possible, it considers to utilize
- FPGA (as heterogeneous computing)



**Code Translation** for FPGA (SPGen, Nymble, FloPoCo) Low-level code for FPGA (VHDL etc.)

> Compilation and Execution on FPGA

# **Our Contributions**

Stochastic Arithmetic Tools Discrete Stochastic Arithmetic (DSA) [21] enables us to estimate rounding errors (i.e., the number of correct digits in the result) with 95% accuracy by executing the code 3 times with random-rounding. DSA is a general scheme applicable for any floatingpoint operations: no special algorithms and no code modification are needed. It is a light-weight approach in terms of

development cost compared to the

performance, usability, and

other numerical verification,

validation methods.

(1) The same code is run several times with the *random rounding* mode (results are rounded up / down with the same probability) (2) Different results are obtained (3) The common part in the different results

is assumed to be a reliable result

## CADNA & SAM (Sorbonne University)

- Applications) [18] is a DSA library for FP16/32/64/128
- OpenMP & MPI and on GPUs with CUDA.
- SAM (Stochastic Arithmetic in Multiprecision) [23] is a DSA library for arbitrary-precision with MPFR.

# CADNA (Control of Accuracy and Debugging for Numerical CADNA can be used on CPUs in Fortran/C/C++ codes with Cadna

## PROMISE (Sorbonne University)

- PROMISE (PRecision OptiMISE) [17] is a tool based on **Delta-Debugging** [24] to automatically tune the precision of floating-point
- The validity of the results is
- PROMISE for arbitraryprecision with MPFR
- Higher precision variables in C/C++ codes checked with CADNA We are going to extend ``-----'

## Precision tuning based on Delta-Debugging

# FPGA as an Arbitrary-Precision Computing Platform

FPGA enables us to implement arbitrary-precision on hardware. High-Level Synthesis (HLS) enables us to program it in OpenCL. However, compiling arbitrary-precision code and obtaining high performance are still challenging.

Intel

PCle x16 PCle x16

V100

IB HDR100

IB HDR100

Stratix10

V100 PCle x16

PLX

PCle x16 V100

IB HDR100

IB HDR100

Stratix10

Cygnus system

Xeon Gold

- SPGen (Stream Processor Generator) [14] is a compiler to generate HW module codes in Verilog-HDL for FPGA from input codes in Stream Processing Description (SPD) Format. The SPD uses a data-flow graph representation, which is suitable for FPGA.
- precision floating-point. Currently,
- pattern, like with graph based data structures.

## **Cygnus** (University of Tsukuba)

- supercomputer system equipped
- University of Tsukuba • Each Stratix 10 FPGA has four external links at 100Gbps. 64 FPGAs make 8x8 2D-Torus network for
- This project targets such a

Heterogeneous computing with FPGA & CPU/GPU is also a challenge

## SPGen (RIKEN)

- It supports FP32 only, but we are going to extend SPGen to support arbitrary-
- there is no FPGA compiler supporting arbitrary-precision.

## **Nymble** (TU Darmstadt, RIKEN)

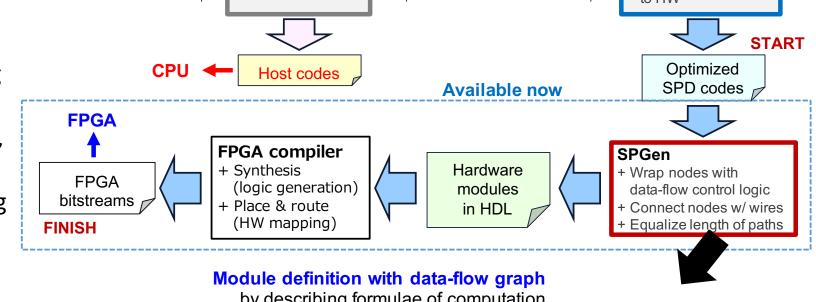
#### • Nymble [15] is another compiler project for FPGA. It directly accepts C codes and has already started to support arbitrary-precision.

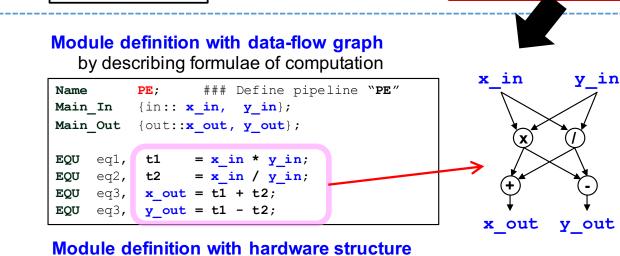
# • It is more suited for non-linear memory access

## Cygnus is the world first with both GPU (4x Tesla V100) and FPGA (2x Stratix 10), installed in CCS,

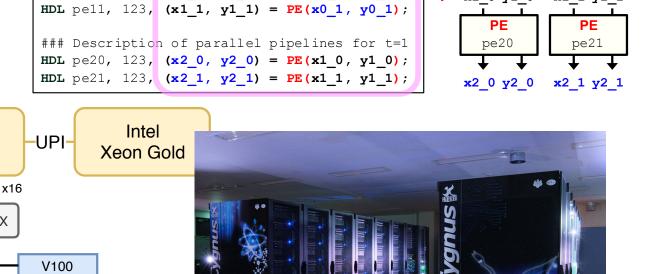
- communication
- heterogeneous system with FPGA.

# C/C++ codes





#### by describing connections of modules Core; ### Define IP core "Core" $x0_0 y0_0 x0_1 y0_1$ Main\_In {in:: x0\_0, x0\_1, y0\_0, y0\_1}; Main\_Out {out::x2\_0, x2\_1, y2\_0, y2\_1}; ### Description of parallel pipelines for t=0 HDL pel0, 123, $(x1_0, y1_0) = PE(x0_0, y0_0);$ $x1_0 y1_0 x1_1 y1_1$



# Fast and Accurate Numerical Libraries

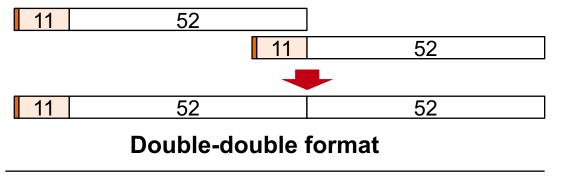
Arbitrary-precision arithmetic is performed using MPFR on CPUs, but the performance is very low. To accelerate it, we are developing several numerical libraries supporting accurate computation based on high-precision arithmetic or algorithmic approach. Some software also support GPU acceleration.

#### QPEigen & QPBLAS (JAEA, RIKEN) Quadruple-precision Eigen solvers (QPEigen) [8, 25] is based on double-double (DD) arithmetic. It is built on a quadruple-precision BLAS (QPBLAS) [9]. They support distributed environments with MPI: equivalent to ScaLAPACK's Eigen solver and

core generator for FPGA

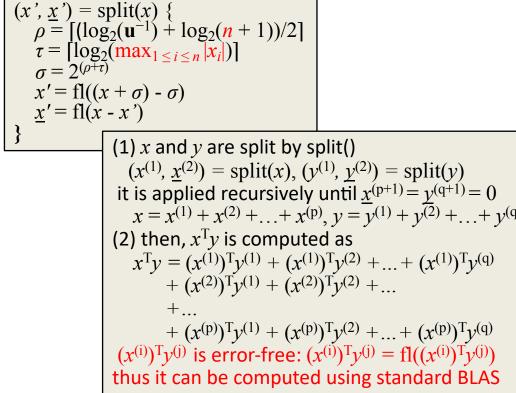
**PBLAS** 

supporting arbitrary-precision.



#### **OzBLAS** (TWCU, RIKEN) OzBLAS [13] is an accurate & reproducible BLAS

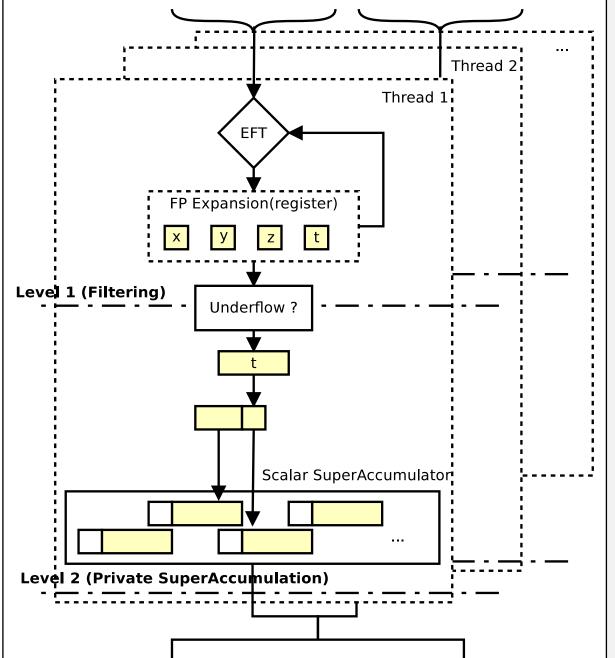
- using Ozaki scheme [18], which is an accurate matrix multiplication method based on the error-free transformation of dot-product
- The accuracy is tunable and depends on the range of the inputs and the vector length CPU and GPU (CUDA) versions



Error-free transformation of dot-product (  $x^{T}y$  )

## **ExBLAS** (Sorbonne University)

- ExBLAS [12] is an accurate & reproducible BLAS
- Assures reproducibility through assuring correctrounding: it preserves every bit of information until



ExBLAS scheme

Level 3 (Parallel Reduction)

Level 4 (Rounding)

- based on floating-point expansions with error-free transformations (EFT: twosum and twoprod) and super-accumulator
- the final rounding to the desired format

CPU (Intel TBB) and GPU (OpenCL) versions

# Conclusion & Future Work

We proposed a new systematic approach for minimal-precision computations. This approach is reliable, general, comprehensive, highperformant, and realistic. Although the proposed system is still in development, it can be constructed by combining already available (developed) in-house technologies as well as extending them. Our ongoing step is to demonstrate the system on a small application.

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# **Precision-Optimizer**