

A review of the book “Numerical methods for roots of polynomials. Part I” by John M. McNamee, Elsevier B. V., 2007 (ISBN-13: 978-0-444-52729-5)

The book deals with the problem of finding roots of polynomials with numerical methods. The author of the book is well-known for his bibliography on roots of polynomials.

In the introduction, the author gives some concrete applications on root-finding and quickly reviews some numerical methods to compute the roots of polynomials. These methods will be detailed in the sequel of the book.

Chapter 1 is devoted to presentation of basic tools used in the numerical methods for finding roots: polynomial evaluation, bounds for roots and the notion of convergence.

In Chapter 2, the author presents the Sturm sequences. It is a powerful tool to count the number of real roots in a given interval. It is based on the computation of gcd. If working in floating-point arithmetic, the concept of gcd is ill-posed and a notion of approximate gcd is necessary. At the end of the chapter, the author presents a method to count the number of complex roots in a rectangle.

Chapter 3 is concerned with real roots by continued fractions. The author covers Fourier and Descartes’ rules as well as Budans and Vincent’s theorems. Some improvements of the previous methods are also presented.

The simultaneous methods for computing roots of polynomials are presented in Chapter 4. These methods makes it possible to compute all the roots at the same time instead of one root at a time. Effects of rounding errors are analyzed. Implementations on parallel computers are also studied.

In Chapter 5, the author studies Newton’s and related methods. Variants for multiple roots are also presented. Newton’s method with interval arithmetic is presented. This makes it possible to get rigorous error bound on the computed roots. Parallel versions are also presented.

The last chapter is devoted to matrix methods. It begins with methods based on the computation of eigenvalues of the classical companion matrix. It goes on with methods using others companion matrices. A important part of this chapter concerns the problem of computing the multiplicities of the roots. Modern methods based on the pejorative manifold are reviewed. Error analysis and sensitivity of the the methods are also discussed using tools like pseudozero sets or pseudospectra.

It is a very interesting book to read. It is clearly written and contains numerous examples that makes the results presented in the book clearer. The book also contains many pointers to efficient programs, softwares and libraries to compute roots of polynomial.

In summary, the book is a handbook of methods for computing roots of polynomials. This book should be a reference to anyone doing research in polynomial roots.

Stef Graillat, Paris, March 2010