Optimizing Precision for High-Performance, Robust, and Energy-Efficient Computations

A joint France-Japan research project

Sorbonne University, CNRS, LIP6, France
Roman Iakymchuk, Stef Graillat, Fabienne Jézéquel

RIKEN Center for Computational Science (R-CCS), Japan
Daichi Mukunoki, Toshiyuki Imamura, Yiyu Tan, Atsushi Koshiba, Jens Huthmann, Kentaro Sano

Center for Computational Sciences, University of Tsukuba, Japan
Norihisa Fujita, Taisuke Boku

* All authors contributed equally to this research

Introduction

In numerical computations, the precision of floating-point computations is a key factor to determine the performance (speed and energy-efficiency) as well as the reliability (accuracy and reproducibility). However, the precision generally plays a contrary role for both. Therefore, the ultimate concept for maximizing both at the same time is the ***optimized/reduced precision computation through precision-tuning, which adjusts the minimal precision for each operation and data***. Several studies have been already conducted for it so far, but the scope of those studies is limited to the precision-tuning alone. Instead, we propose a more broad concept of the optimized/minimal precision and robust (numerically reliable) computing with precision-tuning, involving both hardware and software stack

Available Components

Red: Components developed by us

(1) Precision-tuning with numerical validation based on stochastic arithmetic
- Rounding-errors can be estimated stochastically **with a reasonable cost** (for details, see "(A) Stochastic Arithmetic Tools" at the bottom left)
- General scheme applicable **for any floating-point computations**

Tools:
- **PROMISE** [17] (based on a stochastic arithmetic library, **CADNA** [18]), Verrou [19], etc.

Related works (not validation-based):
- Precimonious [20], GPUMixer [22] etc.

(2) Arbitrary-precision libraries and fast accurate numerical libraries
- Reduced-/mixed-precision with FP16/FP32/FP64 enables us **to improve performance & energy-efficiency**
- High-precision libraries and fast accurate computation methods have been developed **for reliable & reproducible computation**

Tools:
- High-precision arithmetic: binary128 (intel, gcc), QD [1], MPFR [2], ARPREC [3], CAMPARY [4], etc.
- Accurate sum/dot: AccSum/Dot [5], Ozaki-scheme [6], etc.
- Numerical libraries: MPLAPACK [7], **QPEigen** [8], **QPBLAS** [9], XBLAS [10], ReproBLAS [11], **ExBLAS** [12], **OzBLAS** [13], etc.

(3) Field-Programmable Gate Array (FPGA) with High-Level Synthesis (HLS)
- FPGA enables us to implement any operations on hardware, including arbitrary-precision operations
- HLS enables us to use FPGAs through existing programming languages such as C/C++ and OpenCL
- FPGA can be used to perform **arbitrary-precision computations on hardware** efficiently (high-performance and energy-efficient)

Tools:
- Compilers: **SPGen** [14], **Nymble** [15], etc.
- Custom floating-point operation generator: FloPoCo [16], etc.

Our Proposal

Minimal-Precision Computing

Minimal-precision computing is both reliable (aka robust) and sustainable as it ensures the requested accuracy of the result as well as is energy-efficient

High-performance
Performance can be improved through the minimal-precision as well as fast numerical libraries and accelerators

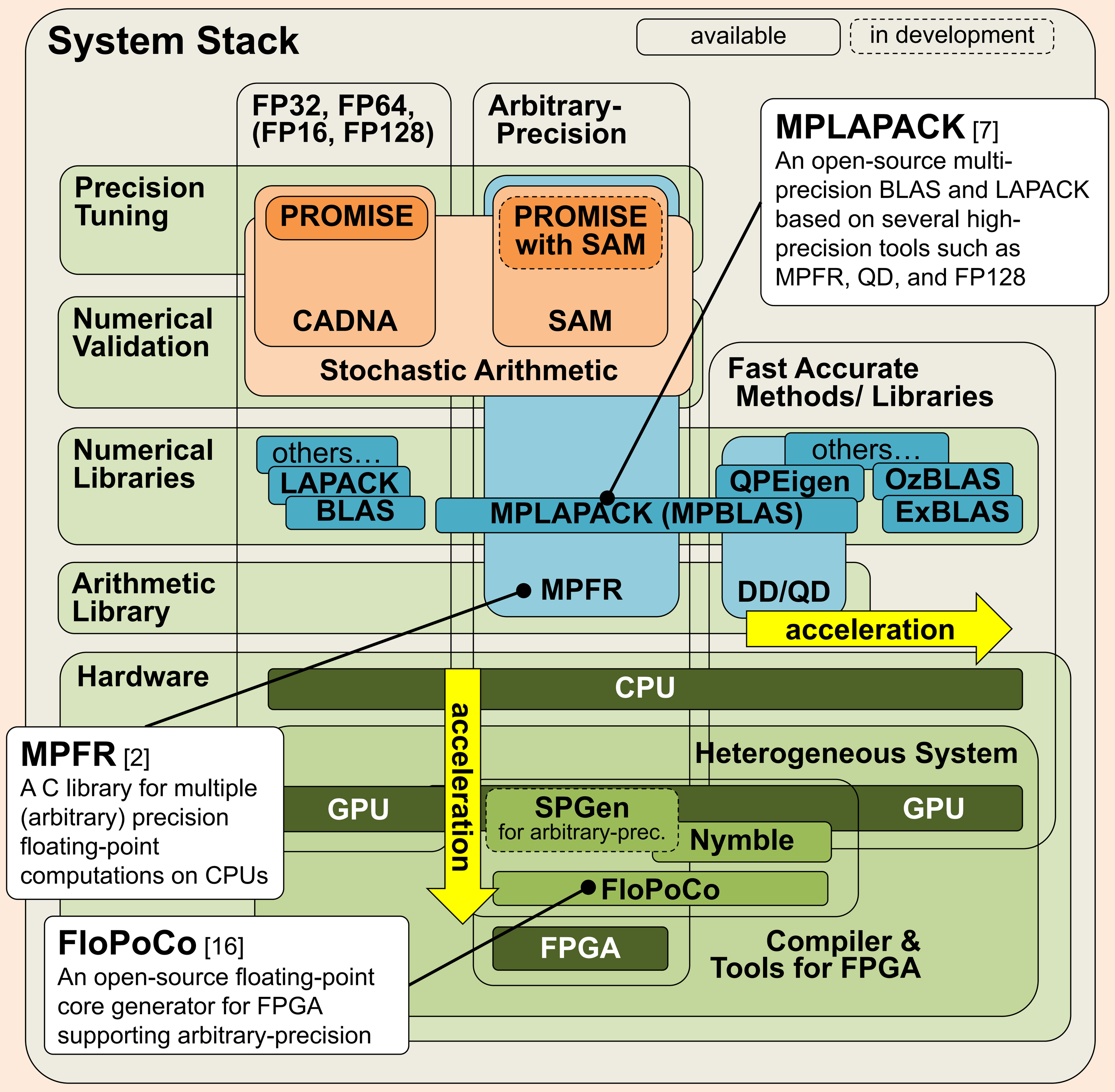
Energy-Efficient
Through the minimal-precision as well as the energy-efficient hardware acceleration with FPGA and GPU

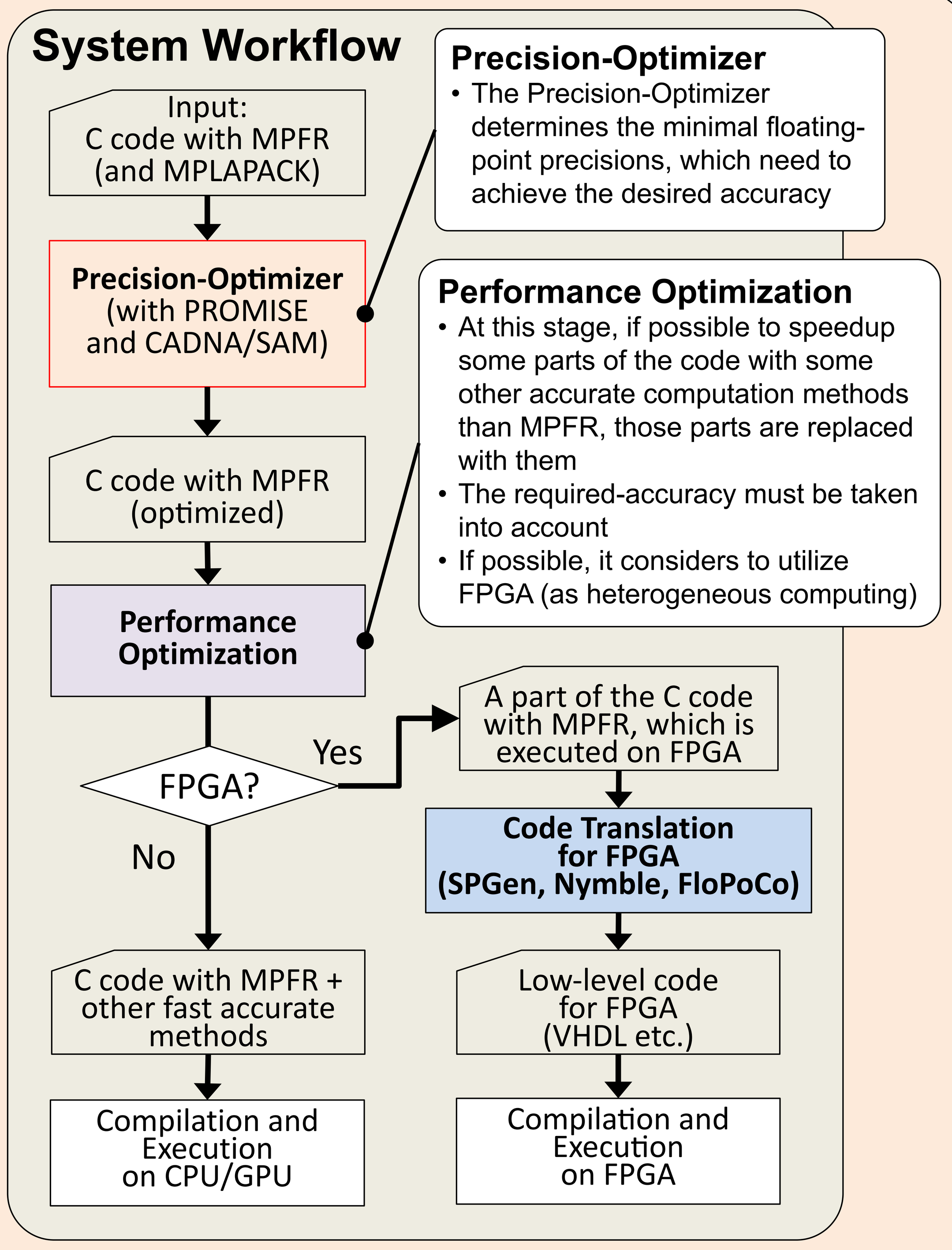
Robust (Numerically Reliable)
To ensure the requested accuracy, the precision-tuning is processed based on numerical validation, guaranteeing also reproducibility

General
Our scheme is applicable for any floating-point computations. It contributes to low development cost and sustainability (easy maintenance and system portability)

Comprehensive
We propose a total system from the precision-tuning to the execution of the tuned code, combining heterogeneous hardware and hierarchical software stack

Realistic
Our system can be realized by combining available in-house technologies





Our Contributions

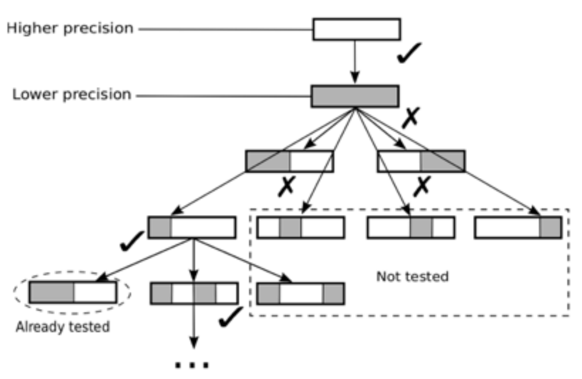
(A) Stochastic Arithmetic Tools

Discrete Stochastic Arithmetic (DSA) [21] enables us to estimate rounding errors (i.e., the number of correct digits in the result) with 95% accuracy by executing the code 3 times with random-rounding. DSA is a general scheme applicable for any floating-point operations: no special algorithms and no code modification are needed. It is a light-weight approach in terms of performance, usability, and development cost compared to the other numerical verification / validation methods.

(1) The same code is run several times with the ***random rounding*** mode (results are rounded up / down with the same probability)
(2) Different results are obtained
(3) The common part in the different results is assumed to be a reliable result

CADNA & SAM (Sorbonne University)
- CADNA (Control of Accuracy and Debugging for Numerical Applications) [18] is a DSA library for FP16/32/64/128
- CADNA can be used on CPUs in Fortran/C/C++ codes with OpenMP & MPI and on GPUs with CUDA.
- SAM (Stochastic Arithmetic in Multiprecision) [23] is a DSA library for arbitrary-precision with MPFR.

PROMISE (Sorbonne University)
- PROMISE (PRecision OptiMISE) [17] is a tool based on **Delta-Debugging** [24] to automatically tune the precision of floating-point variables in C/C++ codes
- The validity of the results is checked with CADNA
- We are going to extend PROMISE for arbitrary-precision with MPFR

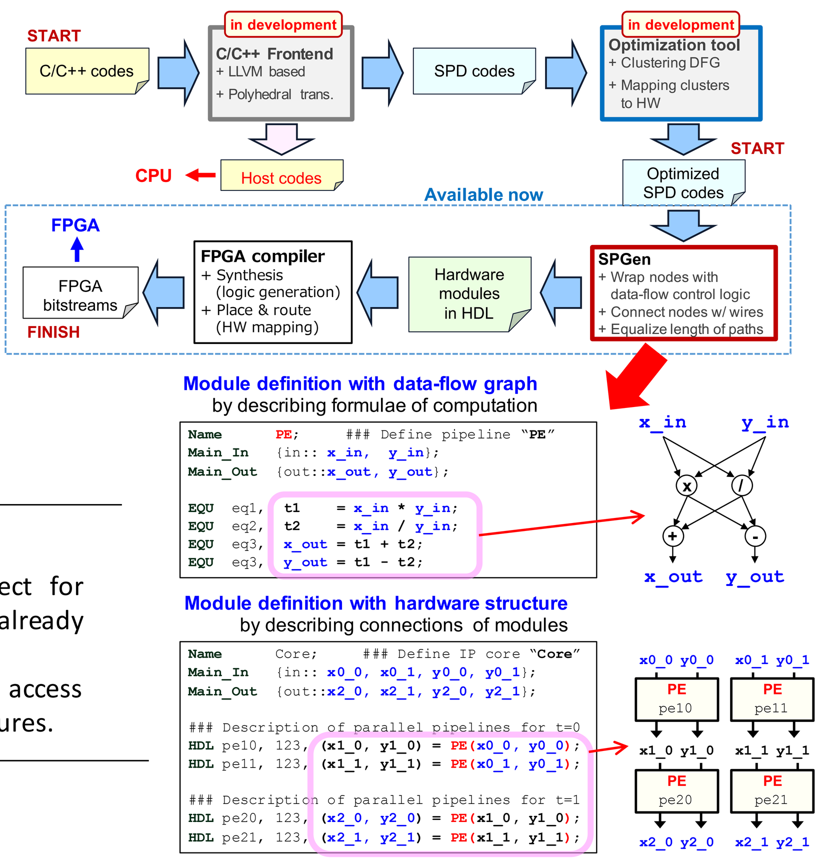


Precision tuning based on Delta-Debugging

(B) FPGA as an Arbitrary-Precision Computing Platform

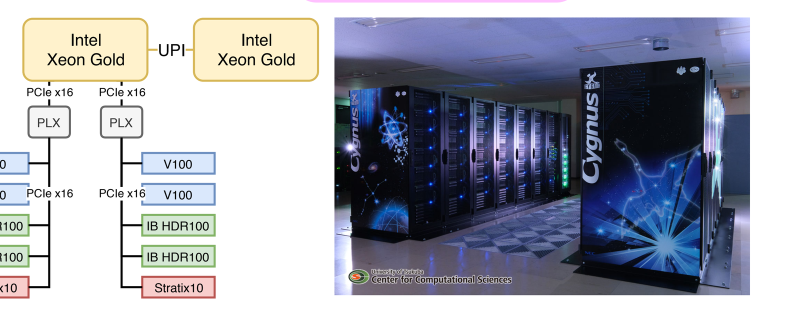
FPGA enables us to implement arbitrary-precision on hardware. High-Level Synthesis (HLS) enables us to program it in OpenCL. However, compiling arbitrary-precision code and obtaining high performance are still challenging. Heterogeneous computing with FPGA & CPU/GPU is also a challenge

SPGen (RIKEN)
- SPGen (Stream Processor Generator) [14] is a compiler to generate HW module codes in Verilog-HDL for FPGA from input codes in Stream Processing Description (SPD) Format. The SPD uses a data-flow graph representation, which is suitable for FPGA.
- It supports FP32 only, but we are going to extend SPGen to support arbitrary-precision floating-point. Currently, there is no FPGA compiler supporting arbitrary-precision.



Nymble (TU Darmstadt, RIKEN)
- Nymble [15] is another compiler project for FPGA. It directly accepts C codes and has already started to support arbitrary-precision.
- It is more suited for non-linear memory access pattern, like with graph based data structures.

Cygnus (University of Tsukuba)
- Cygnus is the world first supercomputer system equipped with both GPU (4x Tesla V100) and FPGA (2x Stratix 10), installed in CCS, University of Tsukuba
- Each Stratix 10 FPGA has four external links at 100Gbps. 64 FPGAs make 8x8 2D-Torus network for communication
- This project targets such a heterogeneous system with FPGA.

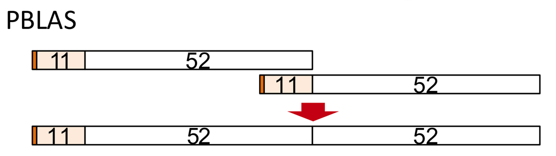


Cygnus system

(C) Fast and Accurate Numerical Libraries

Arbitrary-precision arithmetic is performed using MPFR on CPUs, but the performance is very low. To accelerate it, we are developing several numerical libraries supporting accurate computation based on high-precision arithmetic or algorithmic approach. Some software also support GPU acceleration.

QPEigen & QPBLAS (JAEA, RIKEN)
- Quadruple-precision Eigen solvers (QPEigen) [8, 25] is based on double-double (DD) arithmetic. It is built on a quadruple-precision BLAS (QPBLAS) [9]. They support distributed environments with MPI: equivalent to ScaLAPACK's Eigen solver and PBLAS



Double-double format

OzBLAS (TWCU, RIKEN)
- OzBLAS [13] is an accurate & reproducible BLAS using Ozaki scheme [18], which is an accurate matrix multiplication method based on the error-free transformation of dot-product
- The accuracy is tunable and depends on the range of the inputs and the vector length
- CPU and GPU (CUDA) versions

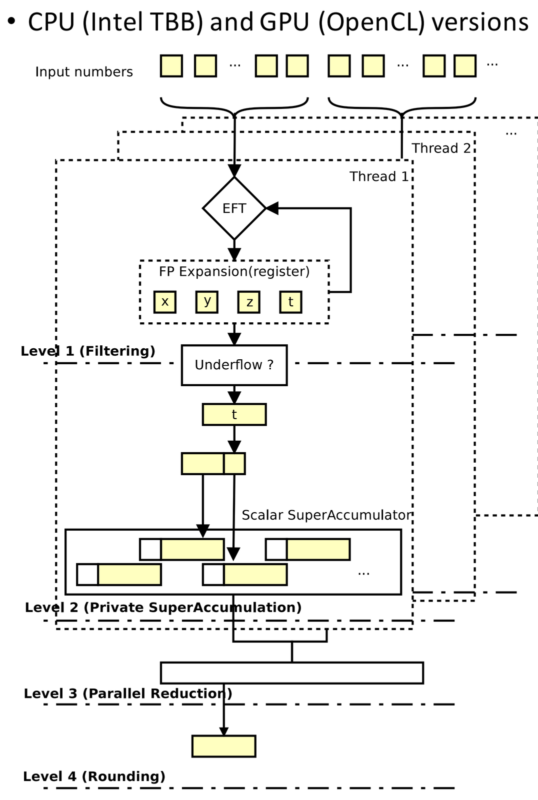
$(x', \underline{x}') = \mathrm{split}(x)$ {
$\rho = \lceil(\log_2(\mathbf{u}^{-1}) + \log_2(n+1))/2\rceil$
$\tau = \lceil\log_2(\max_{1 \le i \le n}|x_i|)\rceil$
$\sigma = 2^{(\rho+\tau)}$
$x' = \mathrm{fl}((x+\sigma) - \sigma)$
$\underline{x}' = \mathrm{fl}(x - x')$
}

Notation:
- fl (…): computation with floating-point arithmetic
- **u**: the unit round-off

(1) x and y are split by split()
$(x^{(1)}, \underline{x}^{(2)}) = \mathrm{split}(x),\ (y^{(1)}, \underline{y}^{(2)}) = \mathrm{split}(y)$
it is applied recursively until $\underline{x}^{(p+1)} = \underline{y}^{(q+1)} = 0$
$x = x^{(1)} + x^{(2)} + \ldots + x^{(p)},\ y = y^{(1)} + y^{(2)} + \ldots + y^{(q)}$
(2) then, $x^T y$ is computed as
$$x^T y = (x^{(1)})^T y^{(1)} + (x^{(1)})^T y^{(2)} + \ldots + (x^{(1)})^T y^{(q)} + (x^{(2)})^T y^{(1)} + (x^{(2)})^T y^{(2)} + \ldots + \ldots + (x^{(p)})^T y^{(1)} + (x^{(p)})^T y^{(2)} + \ldots + (x^{(p)})^T y^{(q)}$$
$(x^{(i)})^T y^{(j)}$ is error-free: $(x^{(i)})^T y^{(j)} = \mathrm{fl}((x^{(i)})^T y^{(j)})$

Error-free transformation of dot-product ($x^T y$)

ExBLAS (Sorbonne University)
- ExBLAS [12] is an accurate & reproducible BLAS based on floating-point expansions with error-free transformations (EFT: twosum and twoprod) and super-accumulator
- Assures reproducibility through assuring correct-rounding: it preserves every bit of information until the final rounding to the desired format
- CPU (Intel TBB) and GPU (OpenCL) versions



ExBLAS scheme

Conclusion & Future Work

We proposed a new systematic approach for minimal-precision computations. This approach is robust, general, comprehensive, high-performant, and realistic. Although the proposed system is still in development, it can be constructed by combining already available (developed) in-house technologies and extending them. Our ongoing step is to demonstrate the system on a proxy application

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