Pseudozero Set of Interval Polynomials

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ABSTRACT

Interval polynomials are useful to describe perturbed polynomials. We present a graphical tool to describe how perturbations of the polynomial coefficients affect its zeros without using interval arithmetic nor matrix representation. This tool implements real pseudozero set that differ from the well known complex pseudozero set restricting perturbations to be real and applied to real polynomials. We introduce a computable formula for this real pseudozero set and compare complex and real pseudozero sets. We propose a graphical MATLAB interface to draw zeros of such interval polynomials.

Categories and Subject Descriptors

G.4 [**Mathematics of Computation**]: Mathematical Software; G.1.5 [**Numerical Analysis**]: Roots of Nonlinear Equations—*Error analysis, Polynomials, methods for*

General Terms

Algorithms, Reliability

Keywords

Polynomial root, pseudozero set, uncertainty, perturbation, interval arithmetic, interval polynomial

1. INTRODUCTION

The computation of polynomial roots is commonly used in several fields of Scientific Computing and Engineering (see for example [1, 14]). In practice, the real or complex polynomial coefficients are often approximate values. Two well known sources of approximation are data uncertainties and rounding errors. The sensitivity of the roots with respect to these perturbations of the polynomial coefficients has been studied with several kinds of methods : condition number, backward error and pseudozero set.

Analytical sensitivity analysis introduces a condition number that bounds the magnitudes of the (first order) changes of the roots with respect to the coefficient perturbations. Numerous results in this direction are available, see for example Gautschi [2] or Wilkinson [15]. When coefficient uncertainty is represented with intervals, interval arithmetic yields over-sets that enclose (sometimes pessimistically) the perturbed roots [7, 6]. Continuous sensitivity analysis, introduced by Ostrowski [12], considers the uncertainty of the coefficients as a continuity problem. The most powerful tool of this last type of methods seems to be the *pseudozero set* of a polynomial on which we focus hereafter. Roughly speaking, it is the set of roots of polynomials that are near to a given polynomial. Pseudozeros were introduced by Mosier [11] but very few applications nor development have been proposed after his work; see also a recent survey by Stetter [14].

Our motivation is to take into account the structure of the perturbations. In this paper, we study two kinds of pseudozero sets. The first one is the *complex pseudozero set*: it is the set of complex numbers that are the roots of polynomials with *complex coefficients* being near to a given polynomial p with *complex coefficients*. The second one is the *real pseudozero set*, that is the set of complex numbers that are the roots of polynomials with *real coefficients* being near to a given polynomial p with *real coefficients*.

For a given polynomial with real coefficients, it makes sense to compute both complex and real pseudozero sets even if the latter may be closer to the physical problem the polynomial represents. This is the case when the polynomial coefficients describe non-complex physical values, such as in transfer function for control theory. Previous works of the authors illustrate how the complex pseudozero set solves stability problems in this area [3, 4]. Another motivation to constrain the pseudozero set to describe real perturbations comes from finite precision computation since the rounding error in real coefficients represented by fixed or floating numbers is always a non-complex perturbation.

Interval real polynomials, *i.e.*, polynomials whose coefficients are real intervals, naturally fit in this last set of perturbed problems. This kind of polynomial arises in robust control theory for modeling uncertainties on the coefficients (see [7]). We use the computable formula to compute real pseudozero set of polynomial to derive a formula to compute the pseudozero set of an interval polynomial.

The paper is organized as follows. In Section 2, we recall some definitions and known results about the complex pseudozero set. In Section 3, we introduce the real pseudozero set and we propose a computable expression for this set. In Section 4, we present pseudozero set of interval polynomials. We relate this set to the real pseudozero set of a given polynomial called the center of this interval polynomial. Then, we present a graphical MATLAB interface to draw such zeros.

2. COMPLEX PSEUDOZERO SET OF POLYNOMIALS

Let $\mathcal{P}_n(\mathbf{C})$ be the linear space of polynomials of degree at most n with complex coefficients ($n \geq 1$). Let p be a polynomial of $\mathcal{P}_n(\mathbf{C})$ with complex coefficients p_i ($0 \leq i \leq n$) such that

$$p(z) = \sum_{i=0}^{n} p_i z^i.$$

We often identify the polynomial p with the vector of its coefficients $(p_0, p_1, \ldots, p_n)^T$. Given a vector $d := (d_0, \ldots, d_n)^T$ in $\mathbf{C}^{n+1}$, we will consider the weighted norm denoted $\|\cdot\|_{\infty,d}$ defined by

$$\|p\|_{\infty,d} = \max_{i=0:n}\{|p_i|/|d_i|\}. \tag{1}$$

The vector d allows freedom in how perturbations are measured.

The real parameter ε bounds the uncertainty in coefficients of p. Given such an $\varepsilon > 0$, a complex ε-neighborhood of p is the set of all polynomials of $\mathcal{P}_n(\mathbf{C})$, close enough to p, that is,

$$N_\varepsilon(p) = \left\{\widehat{p} \in \mathcal{P}_n(\mathbf{C}) : \|p - \widehat{p}\|_{\infty,d} \leq \varepsilon\right\}.$$

The complex ε-pseudozero set of p is defined to include all the zeros of the ε-neighborhood of p. A definition of this set is

$$Z_\varepsilon(p) = \left\{z \in \mathbf{C} : \widehat{p}(z) = 0 \text{ for some } \widehat{p} \in N_\varepsilon(p)\right\}.$$

Theorem 1 below provides a computable counterpart of this definition.

THEOREM 1 (MOSIER [11]). *The complex ε-pseudozero set of p verifies*

$$Z_\varepsilon(p) = \left\{ z \in \mathbf{C} : g(z) := \frac{|p(z)|}{\sum_{i=0}^{n} |d_i||z|^i} \leq \varepsilon \right\}.$$

We recall the proof of [11].

PROOF. If $z \in Z_\varepsilon(p)$ then there exists $\widehat{p} \in \mathcal{P}_n(\mathbf{C})$ such that $\widehat{p}(z) = 0$ and $\|p - \widehat{p}\|_{\infty,d} \leq \varepsilon$. We can note that

$$|p(z)| = |p(z) - \widehat{p}(z)| = \left| \sum_{i=0}^{n} (p_i - \widehat{p}_i) z^i \right| \leq \|p - \widehat{p}\|_{\infty,d} \sum_{i=0}^{n} |d_i||z|^i.$$

Since $\|p - \widehat{p}\|_{\infty,d} \leq \varepsilon$, it follows that

$$\frac{|p(z)|}{\sum_{i=0}^{n} |d_i||z|^i} \leq \varepsilon.$$

Conversely, let $u \in \mathbf{C}$ be such that $|p(u)| \leq \varepsilon \sum_{i=0}^{n} |d_i||z|^i$. If $u \neq 0$, we can write $u = |u|e^{i\theta}$, $\theta \in [0, 2\pi)$ with $|u| > 0$. Let us introduce the polynomial p_u defined by

$$p_u(z) = p(z) - \frac{p(u)}{\sum_{j=0}^{n} |u|^j} \sum_{j=0}^{n} e^{-ij\theta} z^j.$$

A straightforward computation shows that $p_u(u) = 0$ and

$$\|p - p_u\|_{\infty,d} = \frac{|p(u)|}{\sum_{j=0}^{n} |u|^j} \leq \varepsilon.$$

Hence we obtain that u belongs to $Z_\varepsilon(p)$.

If $u = 0$, let us define $p_u(z) = p(z) - p(u)$. It is clear that $p_u(u) = 0$. Besides, we have $\|p - p_u\|_{\infty,d} = |p(u)| \leq \varepsilon$ by hypothesis. In the same way, we get that u belongs to $Z_\varepsilon(p)$. □

This theorem gives us an efficient way to compute the pseudozero set. The ε-pseudozero set of p belong to the interior of the area defined by the contour level (of value ε) of the normalized residual $|p(z)|/\sum_{i=0}^{n} |d_i||z|^i$. MATLAB provides primitives that allow us to easily plot pseudozero set using the following Algorithm 1.

Algorithm 1 Computation of complex ε-pseudozero set (MATLAB version)

Require: polynomial p and uncertainty ε
Ensure: pseudozero set layout in the complex plane
1: We grid a square containing all the roots of p with the MATLAB command `meshgrid`.
2: We compute $g(z)$ at the grid nodes z.
3: We draw the level line $g(z) = \varepsilon$ with the MATLAB command `contourf`.

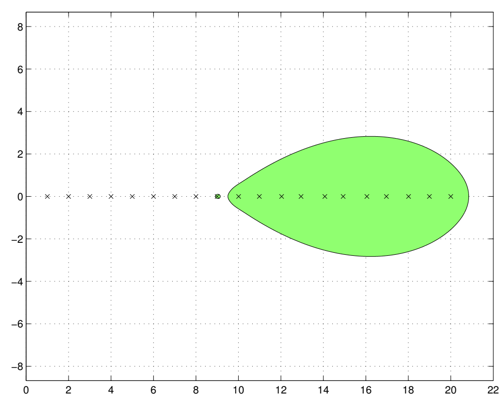

Figure 1: Pseudozero set of Wilkinson polynomial $W_{20} = (z-1)\cdots(z-20)$ when we perturbed only the coefficient z^{19} up to 2^{-23}.

The following proposition proves that each pseudozero component contains at least one root of the polynomial.

PROPOSITION 1 (MOSIER [11, THM. 2]). *Given p in $\mathcal{P}_n(\mathbf{C})$ of degree n, assume that the pseudozero set $Z_\varepsilon(p)$ is bounded. If $q \in N_\varepsilon(p)$, then p and q have the same number of roots, counting multiplicities, in each connected component of $Z_\varepsilon(p)$. Furthermore, there is at least one root of p in each connected component of $Z_\varepsilon(p)$.*

3. REAL PSEUDOZERO SET OF POLYNOMIALS

Now, we introduce the real pseudozero set together with its computable expression.

The notations are similar to the complex case. For $n \geq 1$, let $\mathcal{P}_n(\mathbf{R})$ be the linear space of polynomials of degree at most n with real coefficients. Let p be a polynomial of $\mathcal{P}_n(\mathbf{R})$ with real coefficients p_i such that

$$p(z) = \sum_{i=0}^{n} p_i z^i.$$

We often identify the polynomial p with the vector of its coefficients $(p_0, p_1, \ldots, p_n)^T$. Given a vector $d := (d_0, \ldots, d_n)^T$ in $\mathbf{C}^{n+1}$, we will consider the weighted norm, denoted $\|\cdot\|_{\infty,d}$, defined by

$$\|p\|_{\infty,d} = \max_{i=0:n}\{|p_i|/|d_i|\}.$$

The vector d allows freedom in how perturbations are measured.

Let ε be a given bound of the polynomial coefficient uncertainties. A real ε-neighborhood of p is the set of all polynomials of $\mathcal{P}_n(\mathbf{R})$, close enough to p, that is,

$$N_\varepsilon^R(p) = \{\widehat{p} \in \mathcal{P}_n(\mathbf{R}) : \|p - \widehat{p}\|_{\infty,d} \leq \varepsilon\}. \quad (2)$$

Then the real ε-pseudozero set of p is defined to include all the zeros of the real ε-neighborhood of p. A definition of this set is

$$Z_\varepsilon^R(p) = \left\{z \in \mathbf{C} : \widehat{p}(z) = 0 \text{ for } \widehat{p} \in N_\varepsilon^R(p)\right\}. \quad (3)$$

For $\varepsilon = 0$, the pseudozero set $Z_0^R(p)$ is the set of the roots of p we denote $Z(p)$.

One can easily notice that the real ε-pseudozero set $Z_\varepsilon^R(p)$ is symmetric with respect to the real axis.

Proposition 2. *The real ε-pseudozero set $Z_\varepsilon^R(p)$ is symmetric with respect to the real axis.*

Proof. Let $z \in Z_\varepsilon^R(p)$. It means that there exists $q \in N_\varepsilon^R(p)$ such that $q(z) = 0$. As the polynomial q have real coefficients, this implies that $q(\overline{z}) = 0$. So $\overline{z} \in Z_\varepsilon^R(p)$. $\square$

Following Theorem 2 provides a computable counterpart of this definition. It is based on arguments developed by Hinrichsen and Kelb in [5]. It was proved by Karow [8] using particular perturbations of a companion matrix. Here, we prove this result staying in the field of polynomials. We define for $x, y \in \mathbf{R}^{n+1}$,

$$d(x, \mathbf{R}y) = \inf_{\alpha \in \mathbf{R}} \|x - \alpha y\|_{1,d},$$

the distance of a point $x \in \mathbf{R}^{n+1}$ from the linear subspace $\mathbf{R}y = \{\alpha y, \alpha \in \mathbf{R}\}$. The norm $\|\cdot\|_{1,d}$ is defined for $x \in \mathbf{R}^{n+1}$ by

$$\|x\|_{1,d} := \sum_{i=0}^{n} |d_i||x_i|.$$

Theorem 2. *The real ε-pseudozero set of p verifies*

$$Z_\varepsilon^R(p) = Z(p) \cup \left\{z \in \mathbf{C}\backslash Z(p) : d(G_R(z), \mathbf{R}G_I(z)) \geq \frac{1}{\varepsilon}\right\},$$

where $G_R(z)$ and $G_I(z)$ are the real and imaginary parts of

$$G(z) = \frac{1}{p(z)}(1, z, \ldots, z^n)^T, \ z \in \mathbf{C}\backslash Z(p).$$

In the sequel, we denote by h the function

$$h(z) := d(G_R(z), \mathbf{R}G_I(z)).$$

Proof. Let $z \in Z_\varepsilon^R(p)$. If $p(z) = 0$ then $z \in Z(p)$ else there exists $q \in N_\varepsilon^R(p)$ such that $q(z) = 0$. In this case, we have $p(z) = p(z) - q(z) = (p - q)^T \underline{z}$, where $\underline{z} = (1, z, \ldots, z^n)^T$. It follows that $1 = (p - q)^T G(z)$. Hence we have $1 = (p - q)^T G_R(u) + i(p - q)^T G_I(u)$ and so

$$\begin{cases}(p - q)^T G_R(u) = 1, \\ (p - q)^T G_I(u) = 0.\end{cases}$$

As a consequence, we have $\|p - q\|_{\infty,d}\|G_R(u) - \alpha G_I(u)\|_{1,d} \geq 1$, for all $\alpha \in \mathbf{R}$. We conclude that

$$d(G_R(u), \mathbf{R}G_I(u)) \geq \frac{1}{\|p - q\|_{\infty,d}} \geq \frac{1}{\varepsilon}.$$

Conversely, let $z \in Z(p) \cup \left\{z \in \mathbf{C}\backslash Z(p) : h(z) \geq \frac{1}{\varepsilon}\right\}$. If z belongs to $Z(p)$ then it belongs to $Z_\varepsilon^R(p)$. Otherwise z satisfies $d(G_R(z), \mathbf{R}G_I(z)) \geq 1/\varepsilon$. From a duality theorem (see [9, p.119]), there exists a vector $u \in \mathbf{R}^{n+1}$ with $\|u\|_{\infty,d} = 1$ satisfying

$$u^T G_R(z) = d(G_R(z), \mathbf{R}G_I(z)) \quad \text{and} \quad u^T G_I(z) = 0.$$

Let us consider the real polynomial

$$q = p - \frac{u}{d(G_R(z), \mathbf{R}G_I(z))}.$$

We have

$$\begin{aligned} q(z) &= p(z) - \frac{u^T \underline{z}}{d(G_R(z), \mathbf{R}G_I(z))} \\ &= p(z) - \frac{p(z) u^T G(z)}{d(G_R(z), \mathbf{R}G_I(z))} = 0. \end{aligned}$$

Furthermore we have $\|q - p\|_{\infty,d} = 1/d(G_R(z), \mathbf{R}G_I(z))$. It follows that $\|p - q\|_{\infty,d} \leq \varepsilon$. $\square$

To compute the real ε-pseudozero set $Z_\varepsilon^R(p)$, we only have to evaluate the distance $d(G_R(z), \mathbf{R}G_I(z))$. It is shown in [8, Prop. 7.7.2] that

$$d(x, \mathbf{R}y) = \begin{cases} \min_{\substack{i=0:n \\ y_i \neq 0}} \|x - (x_i/y_i)y\|_{1,d} & \text{if } y \neq 0, \\ \|x\|_{1,d} & \text{if } y = 0. \end{cases}$$

Then the real pseudozero set is also the interior of the area defined by the level contour of function h defined by Relation 2. We can compute the real ε-pseudozero set with following Algorithm 2.

Algorithm 2 Computation of real ε-pseudozero set (MATLAB version)

Require: polynomial p and uncertainty ε
Ensure: pseudozero set layout in the complex plane
1: We grid a square containing all the roots of p with the MATLAB command `meshgrid`.
2: We compute $h(z)$ at the grid nodes z.
3: We draw the level line $h(z) = 1/\varepsilon$ with the MATLAB command `contourf`.

Again MATLAB provides primitives that allow us to plot pseudozero set with Algorithm 2.

The following proposition proves that each pseudozero component contains at least one root of the polynomial.

Proposition 3. *Given p in $\mathcal{P}_n(\mathbf{R})$ of degree n, assume that the pseudozero set $Z_\varepsilon^R(p)$ is bounded. If $q \in N_\varepsilon^R(p)$, then* *p and q have the same number of roots, counting multiplicities, in each connected component of $Z_\varepsilon^R(p)$. Furthermore, there is at least one root of p in each connected component of $Z_\varepsilon^R(p)$.*

The proof is similar to the one for complex pseudozero set.

Stetter [13] proved other detailed results for real pseudozero set.

PROPOSITION 4 (STETTER [13, THM. 3.3]). *Let p in $\mathcal{P}_n(\mathbf{R})$ be a real polynomial of degree n. Let Z_μ be a connected component of the pseudozero set $Z_\varepsilon^R(p)$ such that p has only one root in Z_μ. Then Z_μ satisfies either $Z_\mu \subset \mathbf{R}$ or $Z_\mu \cap \mathbf{R} = \emptyset$.*

As we have seen before, the real pseudozero set is closely related to the function d. This function can have a discontinuous behavior. It is the subject of the following lemma.

LEMMA 1 (HINRICHSEN AND KELB [5]). *The function*

$$d : \mathbf{R}^{n+1} \times \mathbf{R}^{n+1} \to \mathbf{R}_+, \quad (x,y) \mapsto d(x, \mathbf{R}y)$$

is continuous at all pairs (x,y) with $y \neq 0$ or $x = 0$ and discontinuous at all pairs $(x,0) \in \mathbf{R}^{n+1} \times \mathbf{R}^{n+1}$, $x \neq 0$.

This lemma states that a discontinuity problem arises when vector y vanishes. In our case, the discontinuity arises when $G_I(z) = 0$ where G_I is the imaginary part $G_I(z)$ of

$$G(z) = \frac{1}{p(z)}(1, z, \ldots, z^n)^T.$$

It follows that G_I vanishes for $z \in \mathbf{R}$, that is along the real axis. This explains why the contour function of MATLAB may fail and gives some bad results along the real axis. Of course, if none of the zeros of the polynomial is real, the real pseudozero set is correct because we do not evaluate the function G on the real axis.

To avoid this problem, we compute in a specific way the pseudozero set on the real axis. To do this, we use the following lemma.

LEMMA 2. *Being $z \in \mathbf{R}$, z belongs to $Z_\varepsilon^R(p)$ if and only if z belongs to $Z_\varepsilon(p)$ (complex version of the pseudozero set).*

PROOF. This is true because the formula involved in Theorem 1 and in the proof stays in the real field if z is real. □

4. PSEUDOZERO SET OF INTERVAL POLYNOMIALS

4.1 Polynomials with interval coefficients

An *interval polynomial* is a polynomial whose coefficients are real intervals. We denote by $\mathbf{IR}[z]$ the set of interval polynomials and by $\mathbf{IR}_n[z]$ the set of interval polynomials with degree at most n. Let $p \in \mathbf{IR}_n[z]$. It can be written as

$$p(z) = \sum_{i=0}^{n} [a_i, b_i] z^i.$$

The zeros of the interval polynomial is the set (denoted $\mathbf{Z}(p)$) defined by

$$\mathbf{Z}(p) := \{z \in \mathbf{C} : \text{there exist } m_i \in [a_i, b_i], \; i = 0 : n \text{ such that} \sum_{i=0}^{n} m_i z^i = 0\}.$$

We assume in the sequel that the leading interval $[a_n, b_n]$ does not contain 0, because, otherwise, the set $\mathbf{Z}(p)$ become unbounded. Our aim is to compute $\mathbf{Z}(p)$. In order to do this, we need to introduce the center polynomial p^c that satisfies

$$p^c(z) = \sum_{i=0}^{n} c_i z^i,$$

where $c_i = (a_i + b_i)/2$. Let us denote $d_i := (b_i - a_i)/2$.

PROPOSITION 5. *With the notations described above, we have*

$$\mathbf{Z}(p) = Z_\varepsilon^R(p^c) \text{ for } \varepsilon = 1.$$

PROOF. Let $z \in \mathbf{C}$ belonging to $\mathbf{Z}(p)$. It means that there exist $m_i \in [a_i, b_i]$ such that $m(z) = \sum_{i=0}^{n} m_i z^i = 0$. Let p^c the center polynomial defined as above. By definition of p^c, we have

$$\|m_i - p_i^c\|_{\infty,d} = \max_{i=0:n}\{|m_i - p_i^c|/|d_i|\} \leq 1,$$

since $d_i := (b_i - a_i)/2$ and $p_i^c = (b_i + a_i)/2$. It follows that z belongs to $Z_\varepsilon^R(p^c)$ with $\varepsilon = 1$. Conversely, let z belonging to $Z_\varepsilon^R(p^c)$. It means that there exists a polynomial q such that $\|q - p^c\|_{\infty,d} \leq 1$. Then it follows that $\max_{i=0:n}\{|q_i - p_i^c|/|d_i|\} \leq 1$. A simple calculation shows that $a_i \leq q_i \leq b_i$ and so $z \in \mathbf{Z}(p)$. □

Now we have a computable formula to draw pseudozero set of an interval polynomial. The only problem left to deal with is to find a grid that contains the whole pseudozero set. This is solved thanks to the following lemma.

LEMMA 3. *Let $p(z) = \sum_{i=0}^{n} [a_i, b_i] z^i$ be an interval polynomial. Let*

$$R := 1 + \frac{\max_{i=0:n}\{\max\{|a_i|, |b_i|\}\}}{\min\{|a_n|, |b_n|\}}.$$

Then we have,

$$\mathbf{Z}(p) \subset B(O, R),$$

where $B(O,R)$ denotes the ball of the complex plane $\mathbf{C}$ of center O and radius R.

PROOF. Let us denote by $\{z_j\}_{j=1:n}$ the roots of a polynomial p counting with their multiplicities and $r = \max_j |z_j|$. It is well known (see [10, p.154]) that

$$r \leq 1 + \frac{\max\{|p_{n-1}|, |p_{n-2}|, \ldots, |p_0|\}}{|p_n|}. \tag{4}$$

Let z belonging to $\mathbf{Z}(p)$. Then there exists a polynomial $m(z) = \sum_{i=0}^{n} m_i z^i$ such that $m(z) = 0$ and $a_i \leq m_i \leq b_i$ for $i = 0 : n$. It is easy to see that $|m_i| \leq \max\{|a_i|, |b_i|\}$ and $|m_n| \geq \min\{|a_n|, |b_n|\}$. By applying (4), it follows that

$$|z| \leq 1 + \frac{\max_{i=0:n}\{\max\{|a_i|, |b_i|\}\}}{\min\{|a_n|, |b_n|\}}.$$

□

4.2 Presentation of PSIP

PSIP (Pseudozero Set of Interval Polynomials) is a GUI (Graphical User Interface) that integrates MATLAB routines for drawing pseudozero set of interval polynomials.

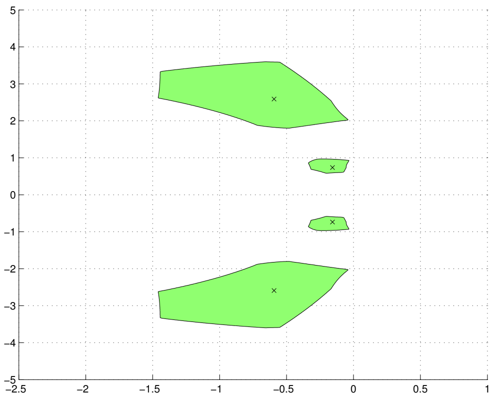

Figure 2: Pseudozero sets of two interval polynomials $p(z) = [1,2]z^4 + [3/2,3]z^3 + [10,14]z^2 + [3,5\sqrt{2}]z + [5,7]$

The code has been written and tested with MATLAB versions 6.5 (R13).

Inputs are the degree and the interval coefficients. This GUI provides interesting facilities to perform a graphical analysis of the pseudozero set, the PSIP output. For example, the user can zoom to any desired box area or manually define it. Of course we suppose the leading coefficient of the interval polynomial does not contain 0, since otherwise, the pseudozero set is not bounded. The previously described discontinuities on the real axis can sometimes generate detail problems when drawing near this real axis.

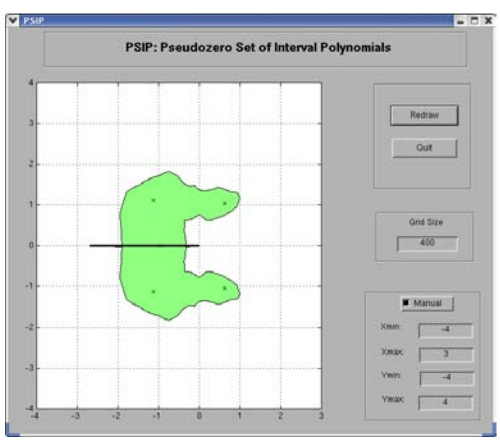

Figure 3: Graphical User Interface of PSIP with the pseudozero set of the interval polynomial $p(z) = z^5 + [1.20, 2.73]z^4 + [1.14, 3.15]z^3 + [0.20, 2.35]z^2 + [1.52, 6.21]z + [0.15, 7.11]$

5. CONCLUSION AND FUTURE WORK

We have proposed a computable formula for the real pseudozero set. Identifying this set is motivated from data errors and rounding errors that corrupt the coefficients of real polynomials. We have applied this formula to compute the pseudozero set of an interval polynomial. The presented Graphical User Interface integrates MATLAB routines to draw such pseudozero sets of interval polynomials and perform qualitative analysis.

An important issue is that we cannot certify the drawing since we use the Matlab command `contourf`. An idea to find an inner and outer approximation of the pseudozero set should be to use the SIVIA algorithm from Jaulin and Walter [7].

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