Minimal-Precision Computing
for High-Performance, Energy-Efficient, and Reliable Computations

A France-Japan joint research project

RIKEN Center for Computational Science (R-CCS) (Japan)
Daichi Mukunoki, Toshiyuki Imamura, Yiyu Tan, Atsushi Koshiba, Jens Huthmann, Kentaro Sano

Sorbonne University, CNRS, LIP6 (France)
Fabienne Jézéquel, Stef Graillat, Roman Iakymchuk

Center for Computational Sciences, University of Tsukuba (Japan)
Norihisa Fujita, Taisuke Boku

* All authors contributed equally to this research.

Introduction

In numerical computations, the precision of floating-point computations is a key factor to determine the performance (speed and energy-efficiency) as well as the reliability (accuracy and reproducibility). However, the precision generally plays a contrary role for both. Therefore, the ultimate concept for maximizing both at the same time is <u>the minimal-precision computation through precision-tuning, which adjusts the optimal precision for each operation and data</u>. Several studies have been already conducted for it so far, but the scope of those studies is limited to the precision-tuning alone. In this study, we propose a more broad concept of the minimal-precision computing with precision-tuning, involving both hardware and software stack.

Available Components

Red: Components developed by us

(1) Precision-tuning with numerical validation based on stochastic arithmetic
- Rounding-errors can be estimated stochastically **with a reasonable cost** (for details, see "A. Stochastic Arithmetic Tools" at the bottom left)
- General scheme applicable **for any floating-point computations**

Tools:
- **PROMISE** [17] (based on a stochastic arithmetic library, **CADNA** [18]), Verrou [19], etc.

Related work (not validation-based):
- Precimonious [20], GPUMixer [22] etc.

(2) Arbitrary-precision libraries and fast accurate numerical libraries
- Reduced-/mixed-precision with FP16/FP32/FP64 enables us **to improve performance & energy-efficiency**
- High-precision libraries and fast accurate computation methods have been developed **for reliable & reproducible computation**

Tools:
- High-precision arithmetic: binary128 (intel, gcc), QD [1], MPFR [2], ARPREC [3], CAMPARY [4], etc.
- Accurate sum/dot: AccSum/Dot [5], Ozaki-scheme [6], etc.
- Numerical libraries: MPLAPACK [7], **QPEigen** [8], **QPBLAS** [9], XBLAS [10], ReproBLAS [11], **ExBLAS** [12], **OzBLAS** [13], etc.

(3) Field-Programmable Gate Array (FPGA) with High-Level Synthesis (HLS)
- FPGA enables us to implement any operations on hardware, including arbitrary-precision operations
- HLS enables us to use FPGAs through existing programming languages such as C/C++ and OpenCL
- FPGA can be used to perform **arbitrary-precision computations on hardware** efficiently (high-performance and energy-efficient)

Tools:
- Compilers: **SPGen** [14], **Nymble** [15], etc.
- Custom floating-point operation generator: FloPoCo [16], etc.

Our Proposal

Minimal-Precision Computing

Minimal-precision computing is both reliable (aka robust) and sustainable as it ensures the requested accuracy of the result as well as is energy-efficient.

High-performance
Performance can be improved through the minimal-precision as well as fast numerical libraries and accelerators

Energy-Efficient
Through the minimal-precision as well as energy efficient hardware acceleration with FPGA and GPU

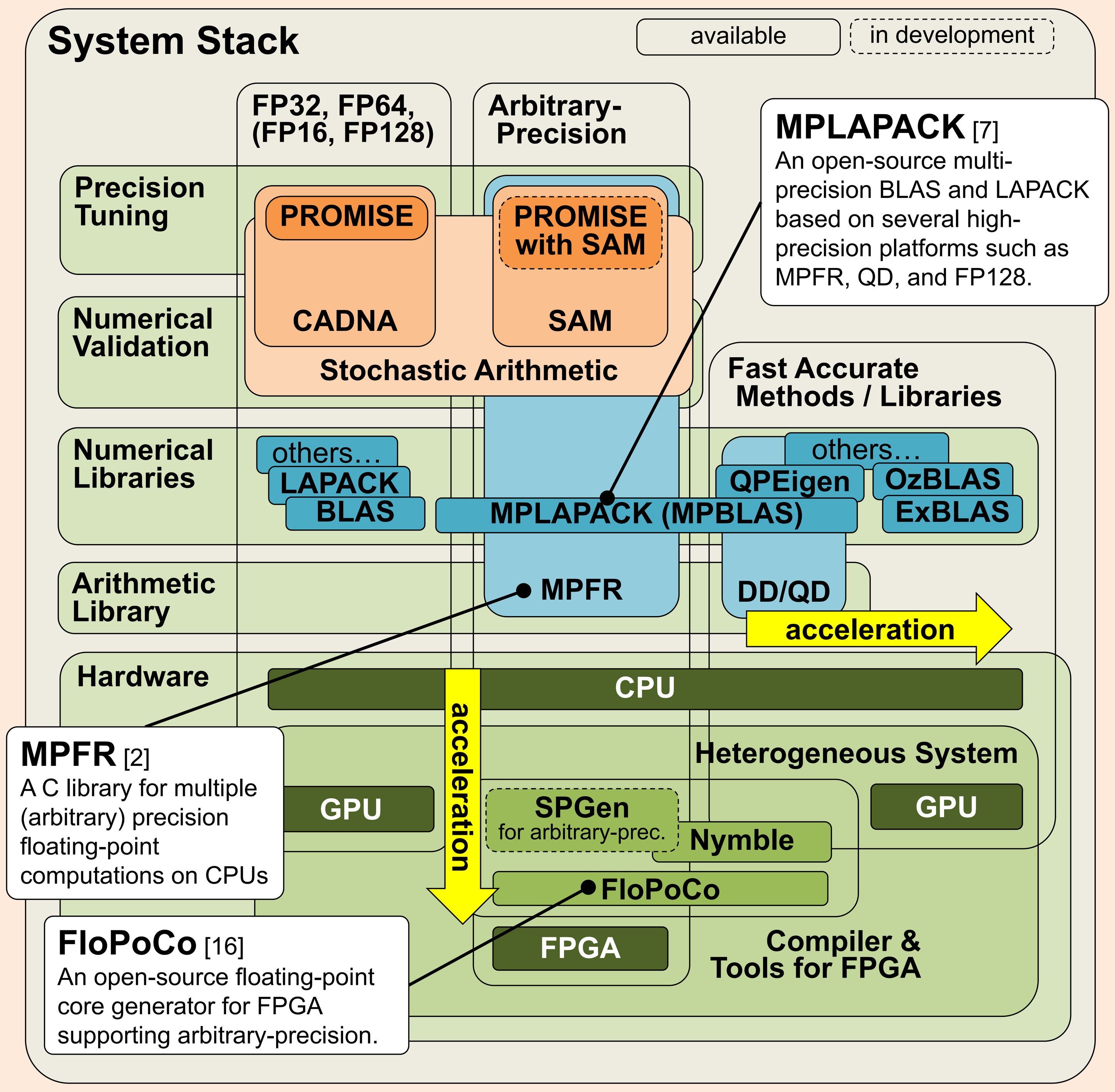
Reliable
To ensure the requested accuracy, the precision-tuning is processed based on numerical validation, guaranteeing also reproducibility

General
Our scheme is applicable for any floating-point computations. It contributes to low development cost and sustainability (easy maintenance and system portability)

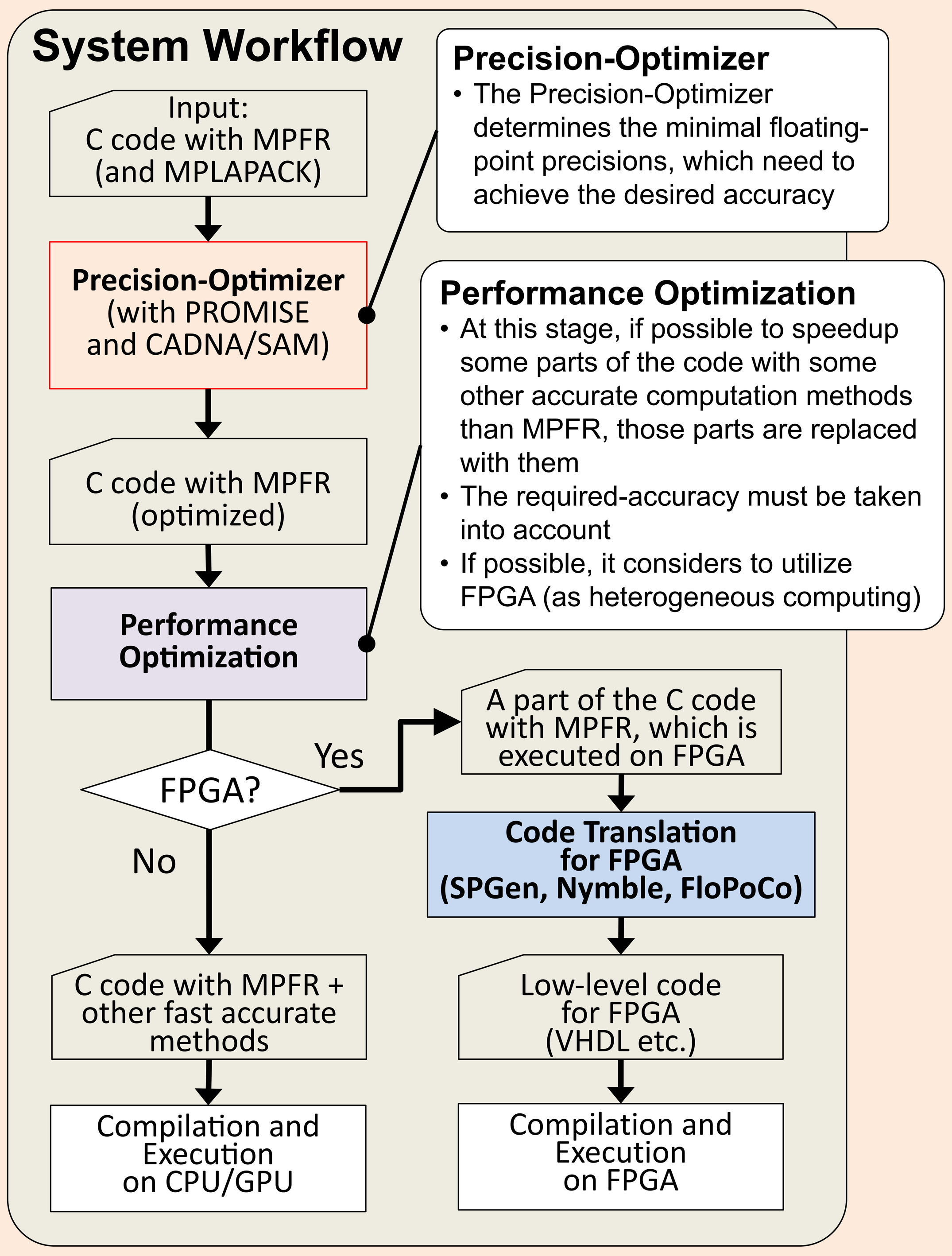
Comprehensive
We propose a total system from the precision-tuning to the execution of the tuned code, combining heterogeneous hardware and hierarchical software stack

Realistic
Our system can be realized by combining available in-house technologies

System Stack



System Workflow



Our Contributions

A. Stochastic Arithmetic Tools

Discrete Stochastic Arithmetic (DSA) [21] enables us to estimate rounding errors (i.e., the number of correct digits in the result) with 95% accuracy by executing the code 3 times with random-rounding. DSA is a general scheme applicable for any floating-point operations: no special algorithms and no code modification are needed. It is a light-weight approach in terms of performance, usability, and development cost compared to the other numerical verification / validation methods.

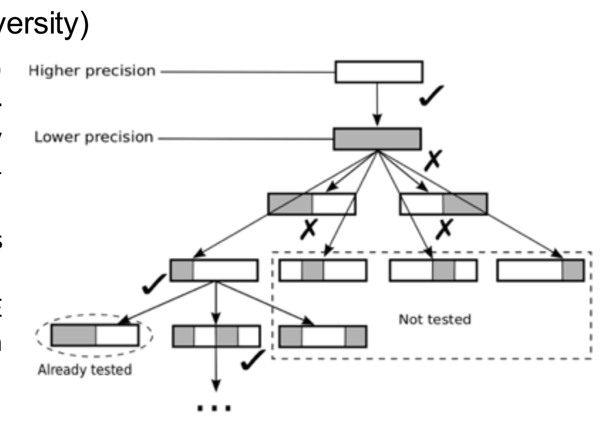
(1) The same code is run several times with the ***random rounding*** mode (results are rounded up / down with the same probability)
(2) Different results are obtained
(3) The common part in the different results is assumed to be a reliable result

CADNA & SAM (Sorbonne University)

- CADNA (Control of Accuracy and Debugging for Numerical Applications) [18] is a DSA library for FP16/32/64/128
- CADNA can be used on CPUs in Fortran/C/C++ codes with OpenMP & MPI and on GPUs with CUDA.
- SAM (Stochastic Arithmetic in Multiprecision) [23] is a DSA library for arbitrary-precision with MPFR.

PROMISE (Sorbonne University)

- PROMISE (PRecision OptiMISE) [17] is a tool based on **Delta-Debugging** [24] to automatically tune the precision of floating-point variables in C/C++ codes
- The validity of the results is checked with CADNA
- We are going to extend PROMISE for arbitrary-precision with MPFR



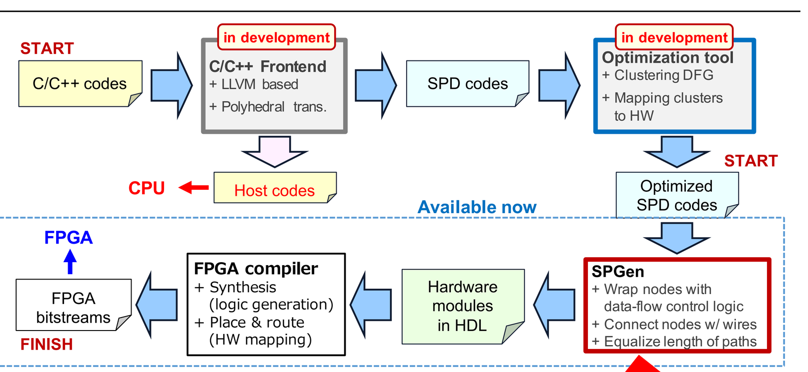
Precision tuning based on Delta-Debugging

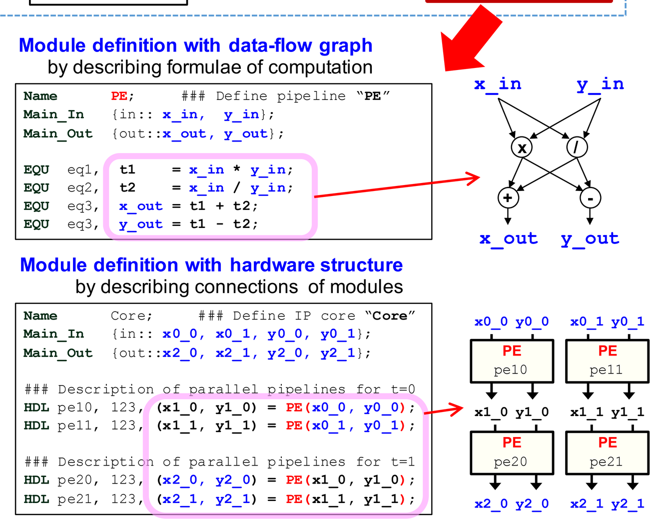
B. FPGA as an Arbitrary-Precision Computing Platform

FPGA enables us to implement arbitrary-precision on hardware. High-Level Synthesis (HLS) enables us to program it in OpenCL. However, compiling arbitrary-precision code and obtaining high performance are still challenging. Heterogeneous computing with FPGA & CPU/GPU is also a challenge

SPGen (RIKEN)

- SPGen (Stream Processor Generator) [14] is a compiler to generate HW module codes in Verilog-HDL for FPGA from input codes in Stream Processing Description (SPD) Format. The SPD uses a data-flow graph representation, which is suitable for FPGA.
- It supports FP32 only, but we are going to extend SPGen to support arbitrary-precision floating-point. Currently, there is no FPGA compiler supporting arbitrary-precision.



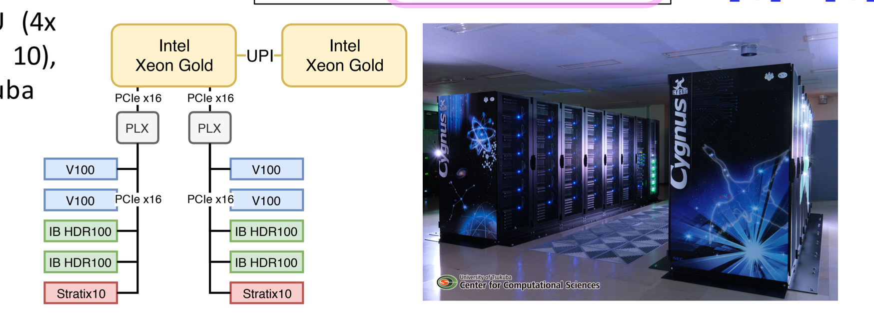


Nymble (TU Darmstadt, RIKEN)

- Nymble [15] is another compiler project for FPGA. It directly accepts C codes and has already started to support arbitrary-precision.
- It is more suited for non-linear memory access pattern, like with graph based data structures.

Cygnus (University of Tsukuba)

- Cygnus is the world first supercomputer system equipped with both GPU (4x Tesla V100) and FPGA (2x Stratix 10), installed in CCS, University of Tsukuba
- Each Stratix 10 FPGA has four external links at 100Gbps. 64 FPGAs make 8x8 2D-Torus network for communication
- This project targets such a heterogeneous system with FPGA.



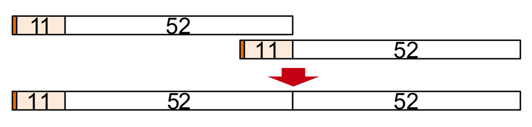
Cygnus system

C. Fast and Accurate Numerical Libraries

Arbitrary-precision arithmetic is performed using MPFR on CPUs, but the performance is very low. To accelerate it, we are developing several numerical libraries supporting accurate computation based on high-precision arithmetic or algorithmic approach. Some software also support GPU acceleration.

QPEigen & QPBLAS (JAEA, RIKEN)

- Quadruple-precision Eigen solvers (QPEigen) [8, 25] is based on double-double (DD) arithmetic. It is built on a quadruple-precision BLAS (QPBLAS) [9]. They support distributed environments with MPI: equivalent to ScaLAPACK's Eigen solver and PBLAS



Double-double format

OzBLAS (TWCU, RIKEN)

- OzBLAS [13] is an accurate & reproducible BLAS using Ozaki scheme [18], which is an accurate matrix multiplication method based on the error-free transformation of dot-product
- The accuracy is tunable and depends on the range of the inputs and the vector length
- CPU and GPU (CUDA) versions

$(x', \underline{x}') = \text{split}(x)$ {
$\rho = \lceil(\log_2(\mathbf{u}^{-1}) + \log_2(n+1))/2\rceil$
$\tau = \lceil\log_2(\max_{1 \le i \le n}|x_i|)\rceil$
$\sigma = 2^{(\rho+\tau)}$
$x' = \text{fl}((x+\sigma) - \sigma)$
$\underline{x}' = \text{fl}(x - x')$
}

Notation:
- fl (…): computation with floating-point arithmetic
- **u**: the unit round-off

(1) x and y are split by split()
$(x^{(1)}, \underline{x}^{(2)}) = \text{split}(x)$, $(y^{(1)}, \underline{y}^{(2)}) = \text{split}(y)$
it is applied recursively until $\underline{x}^{(p+1)} = \underline{y}^{(q+1)} = 0$
$x = x^{(1)} + x^{(2)} + \ldots + x^{(p)}, y = y^{(1)} + y^{(2)} + \ldots + y^{(q)}$
(2) then, x^Ty is computed as

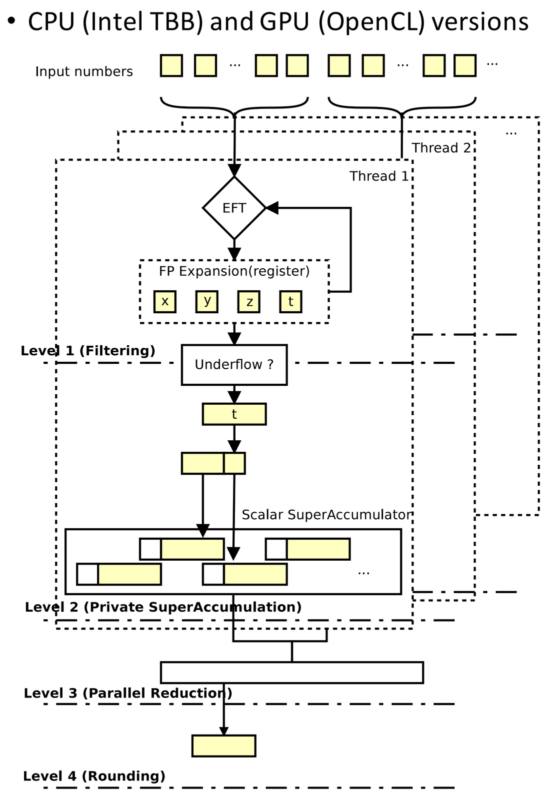
$$x^Ty = (x^{(1)})^Ty^{(1)} + (x^{(1)})^Ty^{(2)} + \ldots + (x^{(1)})^Ty^{(q)} + (x^{(2)})^Ty^{(1)} + (x^{(2)})^Ty^{(2)} + \ldots + \ldots + (x^{(p)})^Ty^{(1)} + (x^{(p)})^Ty^{(2)} + \ldots + (x^{(p)})^Ty^{(q)}$$

$(x^{(i)})^Ty^{(j)}$ is error-free: $(x^{(i)})^Ty^{(j)} = \text{fl}((x^{(i)})^Ty^{(j)})$

Error-free transformation of dot-product (x^Ty)

ExBLAS (Sorbonne University)

- ExBLAS [12] is an accurate & reproducible BLAS based on floating-point expansions with error-free transformations (EFT: twosum and twoprod) and super-accumulator
- Assures reproducibility through assuring correct-rounding: it preserves every bit of information until the final rounding to the desired format
- CPU (Intel TBB) and GPU (OpenCL) versions



ExBLAS scheme

Conclusion & Future Work

We proposed a new systematic approach for minimal-precision computations. This approach is reliable, general, comprehensive, high-performant, and realistic. Although the proposed system is still in development, it can be constructed by combining already available (developed) in-house technologies as well as extending them. Our ongoing step is to demonstrate the system on a small application.

Acknowledgement:
This research was partially supported by the European Union's Horizon 2020 research, innovation programme under the Marie Skłodowska-Curie grant agreement via the Robust project No. 842528, the Japan Society for the Promotion of Science (JSPS) KAKENHI Grant No. 19K20286, and Multidisciplinary Cooperative Research Program in CCS, University of Tsukuba.

References:
[1] D. H. Bailey, "QD (C++/Fortran-90 double-double and quad-double package)," http://crd.lbl.gov/~dhbailey/mpdist
[2] G. Hanrot et al, "MPFR : GNU MPFR Library," http://www.mpfr.org
[3] D. H. Bailey et al., "ARPREC: An Arbitrary Precision Computation Package," Lawrence Berkeley National Laboratory Technical Report, No. LBNL-53651, 2002.
[4] CAMPARY, http://homepages.laas.fr/mmjoldes/campary
[5] T. Ogita et al., "Accurate Sum and Dot Product," SIAM J. Sci. Comput., Vol. 26, pp. 1955-1988, 2005.
[6] K. Ozaki et al., "Error-free transformations of matrix multiplication by using fast routines of matrix multiplication and its applications," Numer. Algorithms, Vol. 59, No. 1, pp. 95-118, 2012.
[7] M. Nakata, "The MPACK (MBLAS/MLAPACK); a multiple precision arithmetic version of BLAS and LAPACK," Ver. 0.6.7, http://mplapack.sourceforge.net, 2010.
[8] Japan Atomic Energy Agency, "Quadruple Precision Eigenvalue Calculation Library QPEigen Ver.1.0 User's Manual," https://ccse.jaea.go.jp/ja/download/qpeigen_k/qpeigen_manual_en-1.0.pdf, 2015.
[9] Japan Atomic Energy Agency, "Quadruple Precision BLAS Routines QPBLAS Ver.1.0 User's Manual," https://ccse.jaea.go.jp/ja/download/qpblas/1.0/qpblas_manual_en-1.0.pdf, 2013.
[10] X. Li et al., "XBLAS – Extra Precise Basic Linear Algebra Subroutines," http://www.netlib.org/xblas
[11] P. Ahrens et al., "ReproBLAS – Reproducible Basic Linear Algebra Sub-programs," https://bebop.cs.berkeley.edu/reproblas in Asia Poster, International Supercomputing Conference (ISC'16), 2016.
[12] R.Iakymchuk et al., "ExBLAS: Reproducible and Accurate BLAS Library," Proc. Numerical Reproducibility at Exascale (NRE2015) workshop at SC15, HAL ID: hal-01202396, 2015.
[13] D. Mukunoki et al., "Accurate and Reproducible BLAS Routines with Ozaki Scheme for Many-core Architectures," Proc. PPAM2019, 2019 (accepted).
[14] K. Sano et al., "Stream Processor Generator for HPC to Embedded Applications on FPGA-based System Platform," Proc. First International Workshop on FPGAs for Software Programmers (FSP 2014), pp. 43-48, 2014.
[15] J. Huthmann et al., "Hardware/software co-compilation with the Nymble system," 2013 8th International Workshop on Reconfigurable and Communication-Centric Systems-on-Chip (ReCoSoC), pp. 1-8, 2013.
[16] F. Dinechin and B. Pasca, "Designing custom arithmetic data paths with FloPoCo," IEEE Design & Test of Computers, Vol. 28, No. 4, pp. 18-27, 2011.
[17] S. Graillat et al., "Auto-tuning for floating-point precision with Discrete Stochastic Arithmetic," Journal of Computational Science (accepted).
[18] F. Jézéquel and J.-M. Chesneaux, "CADNA: a library for estimating round-off error propagation," Computer Physics Communications, Vol. 178, No. 12, pp. 933-955, 2008.
[19] F. Févotte and B. Lathuilière, "Debugging and optimization of HPC programs in mixed precision with the Verrou tool," hal-02044101, 2019.
[20] C. Rubio-González et al., "Precimonious: Tuning Assistant for Floating-Point Precision," Proc. SC'13, 2013.
[21] J. Vignes, "Discrete Stochastic Arithmetic for Validating Results of Numerical Software," Numer. Algorithms, Vol. 37, No. 1-4, pp. 377-390, 2004.
[22] I. Laguna, P. C. Wood, R. Singh, S. Bagchi, "GPUMixer: Performance-Driven Floating-Point Tuning for GPU Scientific Applications," Proc. ISC2019, pp. 227-246, 2019.
[23] S. Graillat, et al., "Stochastic Arithmetic in Multiprecision," Mathematics in Computer Science, Vol. 5, No. 4, pp. 359-375, 2011.
[24] A. Zeller and R. Hildebrandt, "Simplifying and isolating failure-inducing input," IEEE Trans. Softw. Eng., Vol. 28, No. 2, pp. 183-200, 2002.
[25] Y. Hirota et al., "Performance of quadruple precision eigenvalue solver libraries QPEigenK and QPEigenG on the K computer", HPC in Asia Poster, International Supercomputing Conference (ISC'16), 2016.